

# Modeling and Optimization in Traffic Flow Management

*New approaches to achieving, assessing, and optimizing safe and efficient management of our ever-growing civil aircraft traffic aim to improve traffic flow and reduce costs.*

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**ABSTRACT** | Traffic flow management (TFM) allocates the various airport, airspace, and other resources to maintain an efficient traffic flow consistent with safety. TFM is a complex area of research involving the disciplines of operations research, guidance and control, human factors, and software engineering. Hundreds of human operators make TFM decisions that involve tens of thousands of aircraft, en route air traffic control centers, the Federal Aviation Administration's System Command Center, and many airline operation centers. This paper provides an overview of how TFM decisions are made today and challenges facing the system in the future, and reviews modeling and optimization approaches for facilitating system-wide modeling, performance assessments, and system-level optimization of the national airspace system in the presence of both en route and airport capacity constraints.

**KEYWORDS** | Aggregate models; air traffic management; collaborative decision making; complex systems; delay modeling; ground delay; large-scale optimization; linear optimization; performance metrics; traffic flow management; uncertainty; weather impacts

## I. INTRODUCTION

Civil aviation is a vital sector of the U.S. economy. In 2004, air transportation and related industries generated an output of \$1.4 trillion,

employing 12 million people in the United States [1]. The National Airspace System (NAS) provides the infrastructure needed for the operation of civil aviation in the United States and refers to all the hardware, software, and people involved in managing air traffic in the United States. Presently, more than 50,000 commercial flights operate in the U.S. airspace alone on a typical day. The capacity of the airspace is limited by the ability to detect conflicts between aircraft and to resolve them in a safe manner. The NAS barely meets today's traffic demand and results in large delays, especially in the presence of bad weather. The Air Transport Association, a group representing airlines, estimated the cost of delays to airlines in 2005 at \$5.9 billion [2]. Demand for air transportation has seen a sixfold increase in the past 30 years, and estimates call for a strong average growth rate of 4.7% during the next 20 years [3]. This increase in demand will put a further strain on the airports and the airspace, resulting in large delays and a breakdown of airline schedules.

The conflict detection and resolution task is referred to as "separation assurance" and is performed by air traffic controllers with the help of decision tools. Traffic flow management (TFM) is the planning of air traffic to avoid exceeding airport and airspace capacity while making effective use of available capacity. TFM in the

current system is procedure-oriented, done at several levels, and uncoordinated.

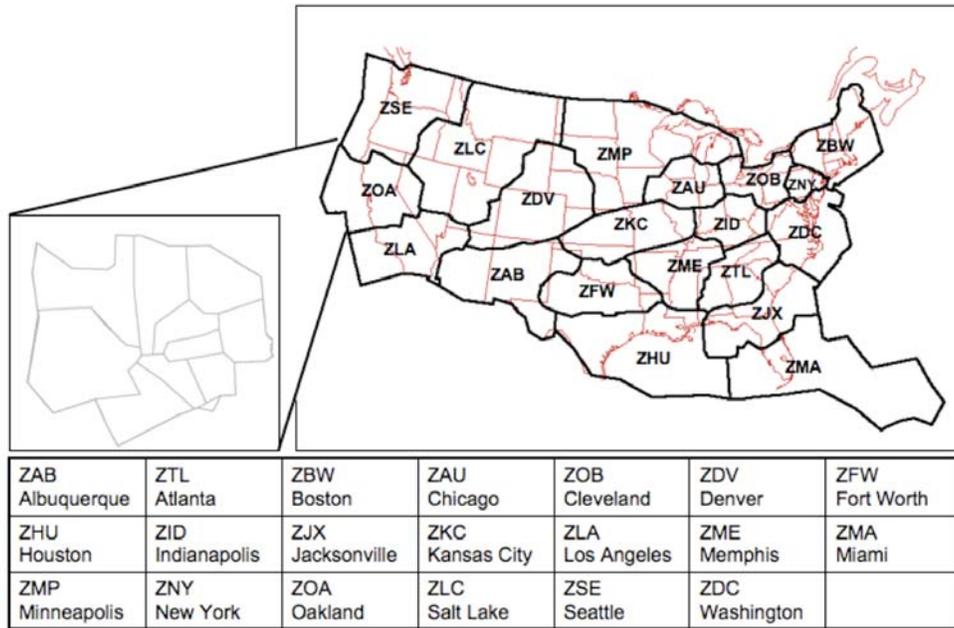
The capacity improvements needed to meet the future demand for air transportation may not be achieved by making incremental changes to the current operations. Along with the enhancements of NAS capacity, improvements are required in the TFM procedures, such as assigning predeparture delays to flights, rerouting, and imposing spacing between aircraft. Major changes are needed with increased levels of automation in a highly safety-oriented environment with new roles for automation, controllers, and pilots. To address the changes required in the air transportation system, the United States has created a multiagency Joint Planning and Development Office (JPDO) to lead the required transformations. To facilitate this transformation, the JPDO is developing a comprehensive concept of operations [4] that advocates operations based on four-dimensional aircraft trajectories. Separation assurance is a safety function that is performed on a time scale of minutes, whereas TFM is an efficiency function that is performed over hours. Both separation assurance and TFM depend on the ability to predict aircraft trajectories and use modeling and optimization to increase the levels of automation to achieve capacity gains.

This paper reviews the algorithms and models that will facilitate system-wide modeling, performance assessments of the NAS, and system-level optimization in the presence of both airspace and airport capacity constraints. Section II provides an overview of TFM under current-day operations and introduces the primary traffic flow management initiatives, personnel involved with decision-making, and key challenges. Since TFM is a complex area of research involving the disciplines of operations research, guidance and control, human factors, and software engineering, the scope of this current paper is stated in the Section II. Recent advances in system-level modeling are discussed in Section III, and metrics

for assessing the performance of the NAS are discussed in Section IV. Approaches for optimizing the flow of traffic in the presence of airport and en route constraints are discussed in Section V, and concluding remarks are provided in Section VI.

## II. OVERVIEW OF TFM TODAY

TFM as practiced today can be viewed as a distributed, hierarchical process. As shown in Fig. 1, the airspace in the United States is divided into 20 centers plus Alaska and Hawaii. The centers are subdivided into sectors [5], [6]. For example, the high-altitude sectors in Oakland Center (ZOA) are depicted in the insert appearing on the left side of Fig. 1. At the top level, the Command Center uses predictions of traffic to form a strategic plan over a 1–24 hr time horizon [7]. Based on the expected weather conditions and demand in the different regions of the airspace and airports, the Command Center may delay some aircraft at airports and/or reroute others. Regional adjustments to these plans are developed by the different centers with a look-ahead time of approximately 20 min to 2 h and take advantage of the reduction in errors due to shorter prediction intervals. These adjustments are done through local rerouting or by spacing aircraft in a stream, referred to as miles-in-trail (MIT). The number of aircraft entering a region is inversely proportional to MIT. MITs are used in increments of 5 mi, and a value ranging from 10 to 30 mi is routinely used to reduce congestion. The current TFM has a hierarchical and distributed control structure and is shown in Fig. 2. Dispatchers and air traffic coordinators at airlines respond to these flow control actions by rescheduling and canceling flights, thus changing flow patterns. Schedules and route preferences from airlines and other users of the system are factored in the development of the TFM strategy through the collaborative decision-making process [8].

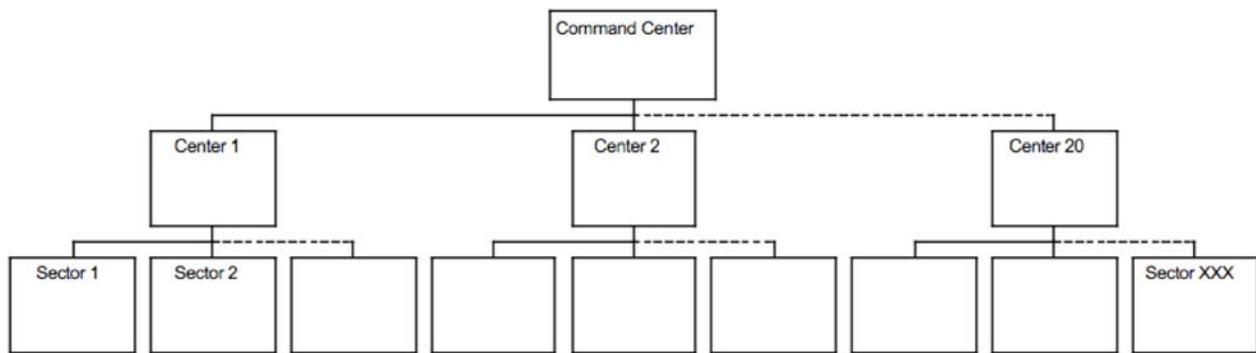


**Fig. 1. Centers in the continental U.S. airspace with sectors shown for Oakland Center.**

Traffic flow management relies on a distributed set of decision makers, each having somewhat disparate goals and information, to control a system characterized by high levels of uncertainty using deterministic procedures. The Command Center is interested in overall flow, the center is interested in the local flow, and the airline operations center is interested in schedule adherence. Each party's decisions are complicated by the inherent uncertainty of the information used to forecast aircraft trajectories and the inability to model the differing objectives and reactions of the other decision makers in a dynamic situation. For example, the traffic forecast does not account for weather uncertainties, departure uncertainties, and

potential airline responses [9].

Severe weather has been identified as the source of 70% of the traffic delays in the United States. Based on FAA operational data, Table 1 shows the performance of the system on four Thursdays in July 2007. The number of flights on all the days is around 24,000. However, the delay increases from 341,431 minutes on a normal day (July 5) to 939,956 minutes on a severe weather day (July 19). Fig. 3 shows the locations of aircraft on a traffic display with aircraft on time shown in gray, aircraft delayed between 15 min and 2 h in blue, and aircraft delayed by more than 2 h in red.



**Fig. 2. Distributed control structure in traffic flow management.**

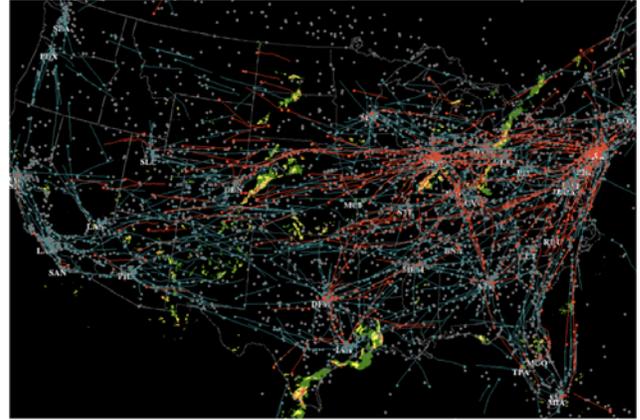
**Table 1 NAS Performance on Normal and Severe Weather Days**

Date	Flight Count	Delayed Flights	Total Delay (min)
07/05/07	23,051	6,094	341,431
07/12/07	24,557	7,129	382,876
07/19/07	24,576	12,114	939,956
07/26/07	24,573	9,606	595,779

The complexity of the TFM problem and the duration of the planning interval lead to a natural decomposition of the problem into national TFM decisions performed at the Command Center providing guidance to centers or groups of centers (regions) and regional TFM decisions performed in the regions. Regional TFM plans are local, more detailed, for shorter durations, and performed using more accurate traffic and weather information.

#### A. National Traffic Flow Management

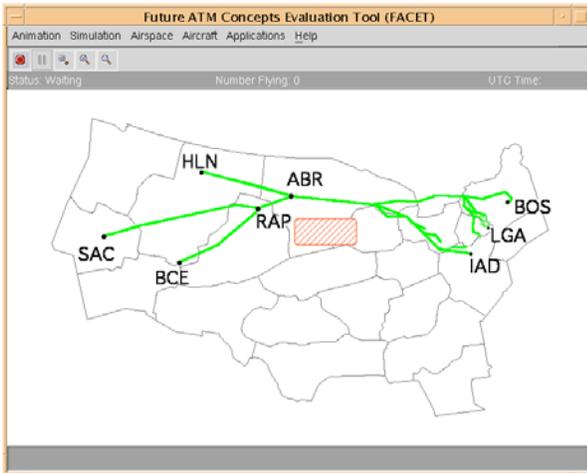
The goal of national traffic flow management is to accommodate user-preferred gate-to-gate trajectory preferences by managing and allocating NAS resources in situations where demand approaches or exceeds supply. The demand and supply situation is made worse during severe weather conditions that may reduce both airspace and airport capacity. The tools available to manage traffic in the presence of excess demand are Airspace Flow Program, ground stop (GS), ground delay programs (GDPs), National Playbook, rerouting, and MIT. Airspace Flow Program identifies flights scheduled to travel through capacity limited regions of airspace, such as a region affected by severe weather. The affected flights are delayed at airports, or the airspace users are provided with the option to route around the constrained regions of airspace. While Airspace Flow Program is used to manage traffic flows due to en route constraints, GDPs and



**Fig. 3. Traffic display with on-time flights in gray, aircraft delayed between 15 min and 2 h in blue, and aircraft delayed more than 2 h in red.**

GSs are used for constraints impacting an airport. Ground stops hold all flights at their departure points that are destined to an affected destination airport for the duration of the ground stop initiative. Like the ground stop, the Ground Delay Program controls the flow of traffic to an airport where the forecasted demand is expected to exceed the airport's predicted acceptance rate. A considerable body of research exists for the optimal planning of ground delay programs under both deterministic and stochastic airport capacity constraints, and a review of this research is presented in Section V.

In addition to imposing departure controls on flights for regulating the flow of traffic into capacity constrained regions of the NAS, routing around these system constraints is a complementary control strategy. Under current operations, the FAA relies on the National Playbook [7]. It is a compendium of standardized alternative routes intended to avoid specific regions of airspace that are commonly affected by severe weather during certain times of the year, based on historically validated data. Playbook also contains alternative routes for circumventing closed airway segments, nonoperational navigational aids, and airports that are impacted by weather or runway closures.



**Fig. 4. Graphical representation of the West Watertown National Playbook route with a conceptual region of severe weather depicted by the shaded polygon.**

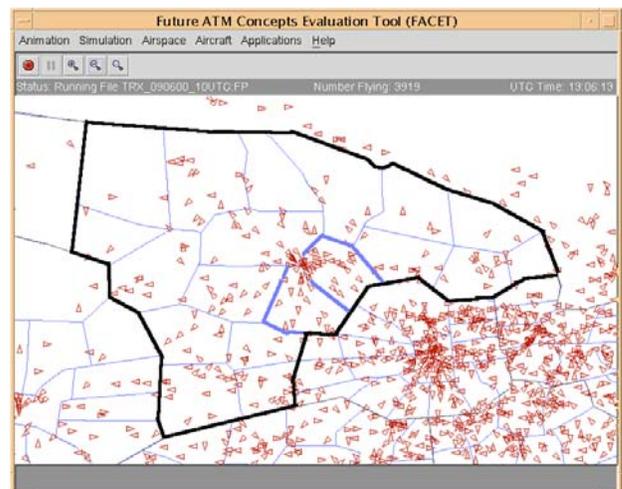
Fig. 4 shows a planning template, known as “West Watertown,” provided in the playbook for rerouting eastbound traffic through the Minneapolis Center when a large portion of airspace in the Midwest is affected by weather. The large rectangular region in the southern portion of the Minneapolis Center (ZMA) represents a predicted severe weather region. The routes represented with a solid line in this figure represent alternative routes for aircraft originating on the West Coast and traveling to select East Coast destinations, such as Boston (BOS), La Guardia (LGA), and Dulles (IAD). Observe from Fig. 4 that the new route avoids the severe weather region entirely.

Visual examination of the West Watertown routes in Fig. 4 shows that the routes from Helena (HLN), Sacramento (SAC), and Bryce Canyon (BCE) merge into a single route at Aberdeen (ABR). By redirecting and merging the usual flows of traffic in a region to avoid severe weather, the playbook-based rerouting process often causes congestion in those regions through which traffic is diverted. This hypothesis was found to be true for the traffic through regions, called Sectors 16 and 17 (shown in blue in Fig. 5), of Minneapolis Center (shown in black in Fig. 5), which was affected by the West Watertown

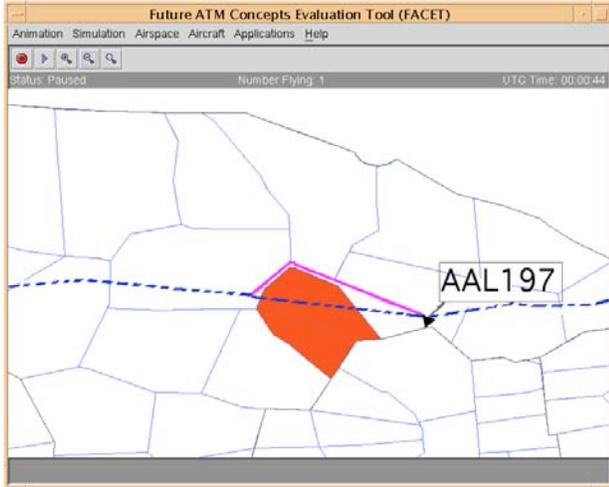
Playbook routes. In the current air traffic management system, traffic flow is reduced by routinely increasing the separation between aircraft, using MIT restrictions, for mitigating this sort of congestion. Application of these restrictions forms the second step of the technique. The example in Section II-B describes a traffic scenario to illustrate the effects of using the West Watertown route along with MIT restrictions to control the traffic volume to within acceptable limits.

### B. Regional Traffic Flow Management

Regional traffic flow management, which operates on a forecasted time horizon of roughly 20 min to 2 h, provides a tactical control loop to adjust the control strategies generated by national flow management based on improved aircraft demand, airspace capacity, and weather intent information. At this time horizon, the two primary flow control strategies are MIT restrictions and local reroutes. MIT imposes a specified interval, expressed in nautical miles, between aircraft in a common stream; however, it tends to overcontrol flights because it is applied to an entire stream of aircraft and lacks the precision needed for minor adjustments. Local rerouting, however, provides the needed flexibility to control a few aircraft to circumvent the congested areas because it builds on the previous solution and obviates the need for



**Fig. 5. Air traffic over Minneapolis Center highlighted in black, with congested sectors 16 and 17 highlighted in blue.**

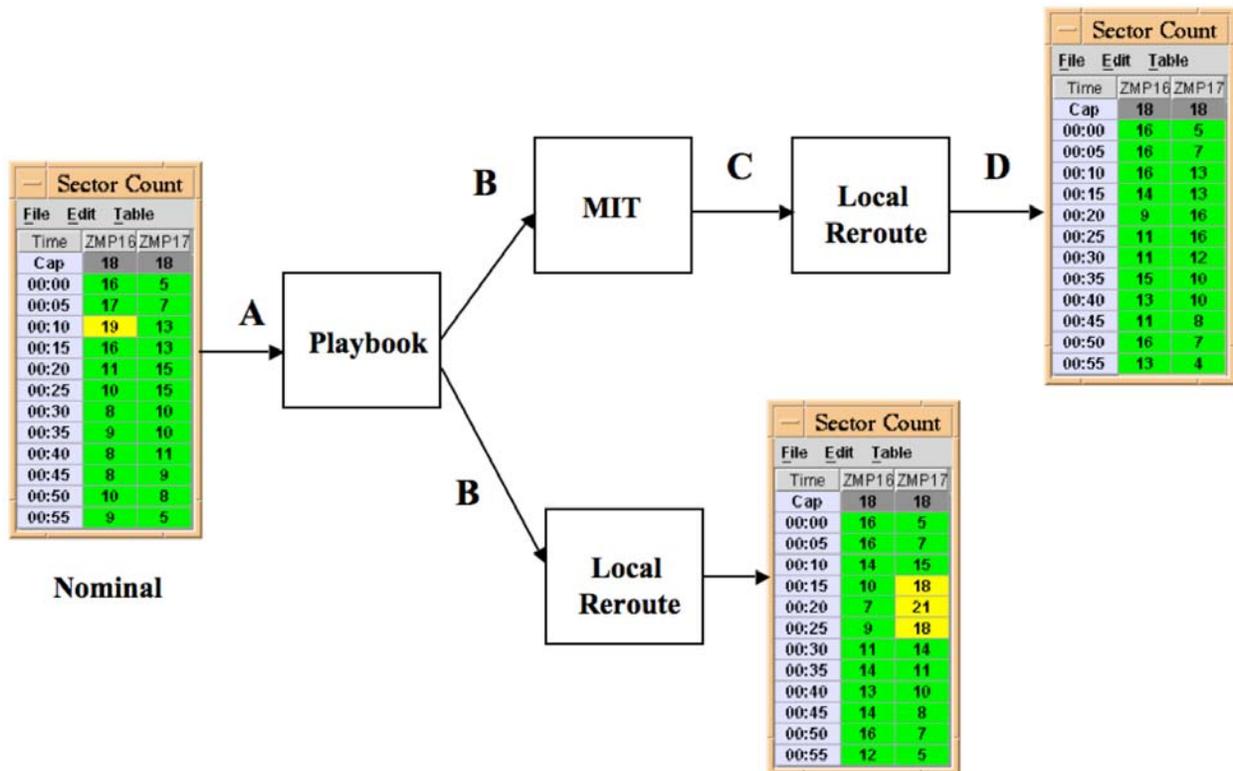


**Fig. 6. Nominal route (dashed blue) and local reroute (magenta) around a capacity constraint (red polygon) in Minneapolis Center.**

a more severe MIT restriction. An example of a local reroute generated using the algorithm described in [10] is shown in Fig. 6. A portion of the nominal route for AAL197, flying from

Boston to San Francisco, is shown as a dashed line. The rerouted path suggested by the algorithm to avoid Sector 16 of Minneapolis Center is shown via a solid line.

Fig. 7 illustrates a potential integrated national and regional flow management solution for the hypothetical weather scenario that is represented by the red polygon in Fig. 4 [10]. Strategically rerouting flights around this constraint using the aforementioned West Watertown National Playbook route leads to an imbalance in the number of flights forecasted to travel through Sectors 16 and 17 in Minneapolis Center (blue highlighted polygons in Fig. 5) and the capacity available in these sectors. As illustrated by Fig. 7, two potential regional-level strategies for mitigating this imbalance are 1) imposing a MIT restriction on the rerouted traffic stream and then tactically rerouting flights within this stream and 2) locally rerouting the traffic without first imposing the MIT restriction. As illustrated by the sector capacity forecasts on the



**Fig. 7. Integrated impact of national and regional level flow control strategies on Minneapolis Sectors in 16 and 17 over a 1-h planning horizon.**

right-hand side of Fig. 7, locally rerouting flights in Minneapolis without first spacing the flights with a MIT restriction results in the capacity-demand imbalance's shifting from Sector 16 to Sector 17.

### C. Issues for Current TFM

Traffic management initiatives such as playbook routes, ground stops, ground delays, and MIT restrictions are based on attempts at solving particular problems. For example, Playbook routes are used for circumventing severe weather, ground stops and ground delays are used for controlling demand at the airports, and MITs are employed for controlling workload in the sectors. The various TFM actions are imposed independently based on experience, and the interaction between different actions may not always be accounted for while making the decisions. The overall capacity of the NAS may be improved by developing methods to integrate and optimize the various traffic management initiatives such as playbook routes, GS, GDP, and MIT to result in a single cohesive plan that improves traffic throughput, reduces delay, reduces congestion, and provides flexibility to the aircraft operators.

A major problem with the current system is the insufficient sharing of information between decision-makers [11]. Information about schedule changes and cancellations is not available in a timely manner to the decision support tools. Airlines are not fully aware of the traffic conditions and the status of the NAS in making their routing decisions. Another source of uncertainty is that most of the trajectory predictions in the crucial climb phase are based on nominal weights of the aircraft and climb procedures [9]. The FAA and the aviation industry have worked hard in recent years to improve the collaboration between users and the service provider in decision-making. Collaborative decision-making (CDM) is limited to strategic planning, and the users' participation in planning reduces as the planning interval becomes smaller [8].

### D. Challenges for Future TFM

A major challenge in the design of future TFM systems is to design an adaptive system that can handle both variations in the magnitude and distribution of the traffic over the next several decades. A system that can handle two to three times current traffic provides a starting point. The new system will be faced with new types of vehicles like very light jets and unmanned air vehicles, and may have to accommodate increased commercial space launch vehicles. The traffic demand may become more variable in the future. The future system should have increased collaboration between the users and the service providers all the way to tactical time frames. Another challenge will be to make good decisions in the presence of uncertainty in the prediction of weather. The effectiveness of probabilistic decision-making should be factored into future TFM. All this can only be achieved through increased levels of automation. The future TFM should gracefully degrade under off-nominal conditions. All future designs must address transition from the current system to the ideal transformed system through a series of intermediate transitions.

### E. Scope of this Paper

The development of a future TFM system requires advances in the disciplines of operations research, guidance and control, human factors, weather prediction, and software engineering. Some of the research may involve the development of new algorithms, while others may focus on the benefits of a certain approach and how easily it can be implemented in the current and future systems based on extensive simulation and field tests. In order to ensure that increased levels of traffic move smoothly and efficiently in the presence of uncertainties, innovative modeling and design methods are needed in TFM. Fig. 8 shows a candidate architecture for integrating the modeling, simulation, and optimization modules to improve decision-making in the future systems. New techniques to minimize total system delay (or other system performance functions) subject to airspace and airport constraints while

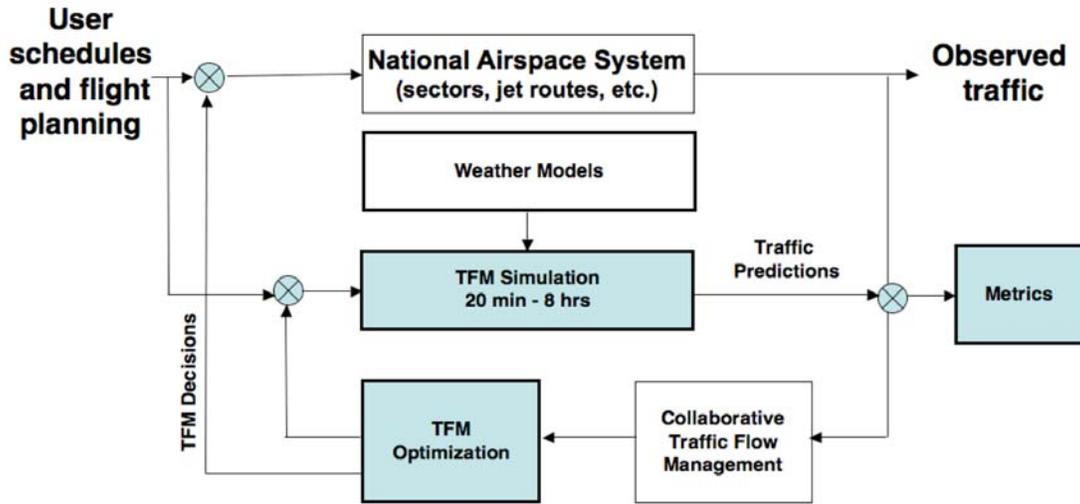


Fig. 8. Potential integrated TFM architecture, with areas covered in this paper highlighted in blue.

accommodating three times traffic in the presence of uncertainty are needed. To keep this paper to a reasonable size, topics were selected both for their importance and the expertise of the authors. This paper reviews the research and state of the art in the following three areas that are highlighted blue in this block diagram: TFM models, delay models, and optimization. For information on recent weather-related air traffic management research, the reader is referred to [12]–[15]. Similarly, recent developments in the area of collaborative traffic flow management are documented in [16] and [17]. The next sections will describe the modeling and optimization techniques useful in realizing the challenges facing future TFM systems while improving the performance of current systems.

### III. TRAFFIC FLOW MODELS

Aggregate and reduced order models simplify the analysis and design of many complex systems. For example, in flight mechanics, design calculations are often made using three degree-of-freedom point mass models that ignore the turn dynamics of the system. Singular perturbation theory is devoted to model order reduction that exploits the inherent time-scales in many systems. Today, air traffic flow prediction is done by propagating the trajectories of the proposed flights forward in time and using them to count the number of aircraft in a

region of the airspace. Examples of systems that use this physics-based modeling approach for demand forecasting include the Center TRACON Automation System [18], the Future ATM Concepts Evaluation Tool (FACET) [19], and the Collaborative Routing Coordination Tool [20]. The accuracy of these predictions is affected by departure and weather uncertainties [9], [21]. These trajectory-based models predict the behavior of the NAS adequately for short durations of up to 20 min. With the short prediction accuracy, it is difficult, if not impossible, to make sound strategic decisions on air traffic management.

Strategic TFM is a hierarchical system consisting of a large number of states and operates over time scales extending from a few hours to 24 h. A strategic TFM decision may involve rerouting all aircraft originating from the West Coast heading to airports on the East Coast to account for an anticipated stormy situation near Chicago over the next several hours. Since strategic TFM requires control of flows of aircraft rather than individual aircraft, an aggregate model of traffic flow that does not use trajectories of individual aircraft is desirable. Strategic TFM can be substantially improved by the development of simpler, but more accurate, models that allow the exploitation of different analysis and synthesis techniques from systems theory. Motivated by this

objective, considerable research has been focused to compute an aggregate model of air traffic flows from historical data.

The development of aggregate flow models for TFM has been the subject of considerable interest since the first model [22] appeared in the literature. This initial model presented a Eulerian approach to modeling air traffic that spatially aggregated the flows in a network of interconnected, one-dimensional control volumes, and leveraged prior research from the area of highway traffic modeling [23]–[25]. The authors subsequently extended their one-dimensional model by modeling the airspace in terms of two-dimensional latitude-longitude surface elements [26]. To account for the inherent flight plan intent uncertainties in the air traffic system, [27] introduced an aggregate stochastic dynamic model that assumed departures from each airport in the NAS were governed by an independent Poisson process. In the actual air traffic management system, the departure rates from airports can vary significantly throughout the day as banks of aircraft arrive and depart from major U.S. hub airports, thus modeling these rates through an independent Poisson processes was viewed as a limitation of this early stochastic dynamic model. To address this issue, [28] developed a linear dynamic systems model that derived actual time-varying departure rates from historical air traffic data that were subsequently augmented by modeling departure uncertainty about these nominal, observed rates.

Subsequent research on aggregate traffic flow models has led to the development of a continuous model [29] based on a modified Lighthill–Whitham–Richards [30] partial-differential equation, as opposed to the earlier discrete models. Finally, to overcome the network splits inherent in the previous models, a multicommodity large-capacity cell transmission model was recently proposed [32]. A brief description of the models proposed in [26], [28], and [29] follows.

### A. Linear Dynamic System Model (LDSM)

The LDSM uses flow relationships between adjacent centers [27]. The model is built by counting the number of aircraft entering a center from an adjacent center, the number of aircraft leaving a center for a neighboring center, and the numbers of aircraft landing and taking off within a center. Input to this model consists of the number of departures. Results presented in [27], assuming that departures follow a Poisson distribution, show that the resulting numbers of aircraft in the centers also fit a Poisson distribution. The main limitation of the results is that modeling departures from Poisson distributions (albeit a different one for each major hub airport) ignores the fact that departure counts vary significantly during the day as banks of aircraft arrive and depart major hub airports. Aircraft counts in the centers, forecast by LDSM, can be improved significantly by accounting for the nominal departure rates as a function of time and augmenting them by modeling departure uncertainty about these nominal rates.

The basic time-invariant LDSM was extended to a time-varying system in [28]. Instead of a single state transition matrix, several state transition matrices (one for each hour) were used to cover the entire prediction period. State transition matrices were computed using historical air traffic data. The resulting model was then driven by average departure rates, also derived from historical air traffic data, to predict aircraft counts in the 23 airspace regions. These 23 regions consisted of 20 centers in the continental United States, one each covering Hawaii and Alaska, and one for the international airspace. Uncertainty bounds around these nominal predictions were then obtained using the standard state covariance propagation model driven by the covariance of departure counts. Day-to-day variations about the average departure counts are assumed to be zero-mean Gaussian random variables. Results are presented for another day of traffic data (other than the four days used in LDSM) to show that these counts lie within the confines of the mean aircraft counts predicted by

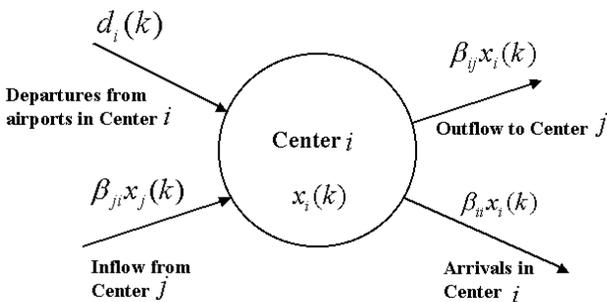
the LDSM and uncertainty bounds generated by the covariance propagation technique [28].

The number of arrivals (landings) and the number of aircraft leaving a center in an interval of time,  $\Delta T$ , are assumed to be proportional to the number of aircraft in the center at the beginning of the interval. Following the notation in Fig. 9 and using the principle of conservation of flow (analogous to the principle of mass balance in a control volume) in a center, the number of aircraft in the center at the next instant of time  $k + 1$  can be related to the number of aircraft in the center at the current instant of time  $k$  via the difference in number of aircraft that came into the center and the number of aircraft that left the center as follows:

$$x_i(k + 1) = x_i(k) - \sum_{j=1}^N \beta_{ij} x_i(k) + \sum_{\substack{j=1 \\ j \neq i}}^N \beta_{ji} x_j(k) + d_i(k) \tag{1}$$

The fractions  $\beta_{ij}$  and  $\beta_{ji}$  are obtained as transition probabilities in [27], [28]. The departures within the center  $i$  are denoted by  $d_i(k)$ . For modeling, these departures can be split into two components—a deterministic one and a stochastic one. The deterministic portion of the departures  $u_i(k)$  can be computed from filed flight plans and from historical departure data. For example,  $u_i(k)$  can be set to the average departure count derived from historical data.

The stochastic component of the departures,



**Fig. 9. Components of aircraft flow contributing to the traffic count in a given center.**

$w_i(k)$ , can be modeled by assuming a suitable distribution such as a Gaussian or a Poisson distribution. In such a model,  $w_i(k)$ , which can also be obtained from historical data, represents the expected variation around the deterministic component.

The discrete system in (1) can be rewritten in the standard state space notation as

$$\mathbf{x}(k + 1) = \mathbf{A}(k)\mathbf{x}(k) + \mathbf{B}(k)\mathbf{u}(k) + \mathbf{C}(k)\mathbf{w}(k) \tag{2}$$

where,

$k$  time instant defined by  $k \Delta T$ , with  $\Delta T$  being the sampling interval. In [28], it has been shown that a 10-min sampling interval accurately approximates center aircraft count.

$\mathbf{x}(k) = [x_1(k), \dots, x_N(k)]$  state vector with the number of aircraft in the centers at time  $k$  as its elements.

$\mathbf{u}(k) = [u_1(k), \dots, u_N(k)]$  control vector with the number of aircraft departing (taking off) from the centers as its elements.

$\mathbf{w}(k) = [w_1(k), \dots, w_N(k)]$  vector for modeling departure uncertainties.

$\mathbf{A}(k)$  state transition matrix that contains the information of how flights transition from one center to the other center.

The elements of the state transition matrix  $A$  are given by

$$a_{ij} = \beta_{ij}; \tag{3}$$

$$i \neq j; i = 1, \dots, N; j = 1, \dots, N$$

where  $N = 23$  is the number of centers. The off-diagonal terms  $a_{ij}(k)$  represent the fraction of aircraft transitioning from center  $i$  to center  $j$  at time  $k$ . This quantity is calculated from historical data and slowly varies over time [28].

The diagonal terms can be calculated as

$$a_{ii} = 1 - \sum_{j=1}^N \beta_{ij} \quad (4)$$

These terms represent the fraction of the aircraft that remained in center  $i$  during the  $k^{\text{th}}$  time step.

Numerical results in [28] provide error bounds for the number of aircraft in the center and show that a linear dynamic system with a few transition matrices and Gaussian departure distribution adequately represents traffic behavior at the Center level.

## B. Eulerian Models

The development of a Eulerian [23]–[25] approach to model air traffic was discussed in recent research efforts [22], [26]. A computer-aided methodology for deriving Eulerian models of the airspace, and employing it for air traffic flow control, is described in [26]. The approach uses FACET software [19] as its foundation.

The Eulerian approach models the airspace in terms of line elements approximating airways, together with merge and diverge nodes. Since this modeling technique spatially aggregates the air traffic, the order of the airspace model depends only on the number of line elements used to represent the airways and not on the number of aircraft operating in the airspace. Eulerian models are in the form of linear, time-varying difference equations.

The one-dimensional modeling methodology is an intuitive approach for deriving models of traffic flow networks formed by jet routes and Victor airways. However, not all aircraft in the airspace strictly follow the jet routes or Victor airways. This introduces the need for a more flexible modeling framework. This framework, first advanced in [22], discretizes the airspace into surface elements, within which the traffic flow is

aggregated into eight different directions. This modeling provides adequate fidelity in en route airspace where the traffic flow is largely two-dimensional. The traffic at all flight levels in Class A airspace (at or above 18,000 ft) is classified as belonging to any one of these eight directions, with inflows and outflows from airports and other external sources. Each surface element is connected to its eight neighbors, with the connection strengths being determined by the actual traffic flow patterns.

## C. Partial Differential Equation Model

In [29], a fully continuous Eulerian model that relies on a modified version of the Lighthill-Whitham-Richards [30] partial differential equation is proposed. Development of this model begins by subdividing the airspace into a series of  $N$  links of length  $L_i$ , described by a coordinate  $x_i \in [0, L_i]$ , where the links are indexed by  $i \in \{1, \dots, N\}$ . If the density of aircraft on link  $i$  at time  $t$  is denoted by  $\rho_i(x_i, t)$  and the main velocity profile along link  $i$  is  $v_i(x_i)$  the governing partial differential equation system is given by [30]

$$\frac{\partial \rho_i(x_i, t)}{\partial t} + \frac{\partial}{\partial x_i} (\rho_i(x_i, t) v_i(x_i)) = 0 \quad \forall i \in \{1, \dots, N\} \quad (5)$$

$$\rho_i(x_i, 0) = \rho_i^{\circ}(x_i) \quad \forall i \in \{1, \dots, N\} \quad (6)$$

$$\rho_i(0, t) v_i(0, t) = \sum_{j \in U(i)} \rho_j(L_j, t) v_j(L_j, t) + q_i^{\text{in}}(t) \quad \forall i \in M \quad (7)$$

$$\rho_i(0, t) v_i(0, t) = \beta_i(t) \rho_i(L_i, t) v_i(L_i, t) \quad \forall i \in D \quad (8)$$

$$\rho_i(0, t) v_i(0, t) = (1 - \beta_i(t)) \rho_i(L_i, t) v_i(L_i, t) \quad \forall i \in D \quad (9)$$

$$\rho_i(0, t) v_i(0, t) = q_i^{\text{in}}(t) \quad \forall i \in S \quad (10)$$

where

$N$	number of links;
$S$	set of source links;
$M$	set of merge links;
$D$	set of fork links;

$U(i)$	set of links merging into link $i$ ;
$i_l, i_r$	indexes of the links of a fork;
$\rho_i^o(x_i, t)$	initial aircraft density on link $i$ ;
$q_i^{in}(t)$	inflow at $x_i = 0$ for link $i$ ;
$\beta_i(t)$	portion of $\rho_i$ that flows into link $i_l$ .

This model was validated by comparing the aircraft count predictions from the model with the actual aircraft counts that were obtained from historical Enhanced Traffic Management System (ETMS) data. The model was subsequently included in an optimization problem that was designed to maximize the throughput at a destination airport subject to en route sector capacity constraints.

#### D. Status of Flow Models

The aggregate and reduced order flow models described earlier represent the traffic behavior to a high degree of accuracy and can be tailored to the time-scales and regions of interest. A comparison of the characteristics of the different flow models can be found in [31] and [32]. The stability and response characteristics of the aggregate flow models are presented in [33]. The aggregation in the flow models generally results in the loss of information about the route structure of individual aircraft. This additional information can be modeled at the expense of the size of the aggregate model. The flow models can be used to design control policies to achieve the desired closed-loop behavior. However, additional research is needed to translate the control policies into actual TFM flight planning decisions involving aircraft departure times and routes. Currently, there is limited experience in the application of aggregate and reduced order flow models for generating TFM decisions. TFM decisions based on aircraft level models and optimizing system level cost functions are described in Section V.

## IV. DELAY MODELS

In order to evaluate the impact of TFM procedures on overall NAS performance, it is necessary to

characterize and model various performance metrics. The performance of the NAS can be described in several different ways. A number of metrics, such as delay, safety, predictability, access, flexibility, and efficiency, have been proposed to describe the performance of the system [34], and details about the metrics are available in [35] and [36]. A correct set of metrics, meaningful and measurable, needs to be created from these desirable performance characteristics in order to understand the complex relation among traffic, weather, and delay. Since no physical model is available for assessing the performance of the TFM system, most delay models leverage the vast quantities of data that are available for the system.

Delay has been used extensively to assess the performance of NAS. An aircraft may be delayed at the departure airport due to problems associated with the aircraft or excessive traffic or weather conditions. These types of delays could also occur during en route and arrival phases of the aircraft. The FAA maintains two databases, Air Traffic Operations Network (OPSNET) and Aviation System Performance Metrics (ASPM).<sup>1</sup> The databases contain information about different types of delay, location and causes of delay, and flight performance relative to schedule. In addition to the delays in the databases, delay can be computed as excess over an ideal time of travel between the origin and destination [37]. The OPSNET delays are caused by the application of TFM initiatives in response to weather conditions and excessive traffic volume.

TFM initiatives such as ground stops, ground delay programs, rerouting, airborne holding, and miles-in-trail restrictions are actions that are needed to control the air traffic demand to mitigate the demand-capacity imbalance due to the reduction in capacity. Consequently, TFM initiatives result in NAS delays. Of all the causes, weather has been identified as the most important causal factor for NAS delays. Therefore, to guide flow control decisions during the day of operations, and for postoperations analysis, it is

<sup>1</sup> Both found at <http://www.apo.data.faa.gov>.

useful to create a baseline for NAS performance and establish a model that characterizes the relation between weather and NAS delays. In postoperations analysis the model can be used to check if the recorded delay was within the range of delays for similar weather and, if the delay is out of bounds, to examine the operations carefully for other causes. Similarly, given the expected weather, the model can be used to predict the expected aggregate delay.

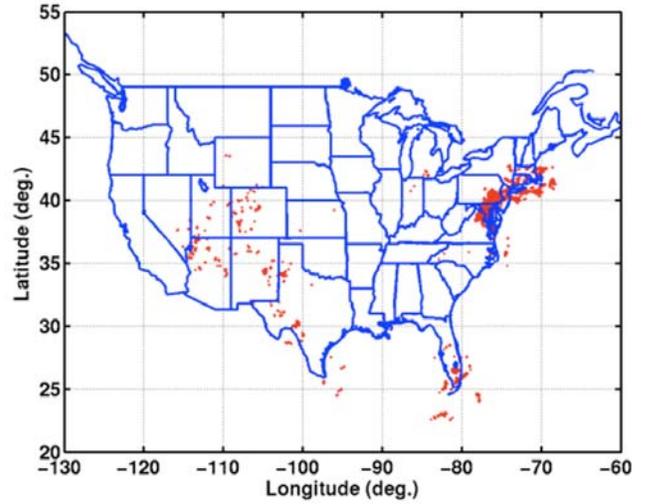
Recent delay estimation models that have appeared in the literature include a sizeable number of models based on the Weather Impacted Traffic Index (WITI) [38]–[42]; passenger delay estimation models [43]; and models that explore the propagation of delays throughout the NAS [44]–[47]. A brief description of recent advances in each of these areas follows.

**A. Weather Impacted Traffic Index Models**

Several efforts have been made to understand the connection between weather and delay at both the local and the national level. The concept of WITI, which is the number of aircraft affected by convective weather at a given instant of time, was introduced in [38] and has subsequently been the basis for the development of several delay estimation models [39]–[42]. Closely paralleling these WITI-based models has been the development of grid-based models that borrow from such fields as computer vision and image processing [48].

WITI is an indicator of the number of aircraft affected by weather. The computation of WITI consists of 1) assigning a value of one to every grid cell  $W_{i,j}$  of the weather grid  $W$ , where severe weather is indicated and zero elsewhere; 2) counting the number of aircraft in every grid cell  $T_{i,j}$ ; and 3) computing the WITI at an instant of time  $k$  (typically at 1-min intervals) as follows:

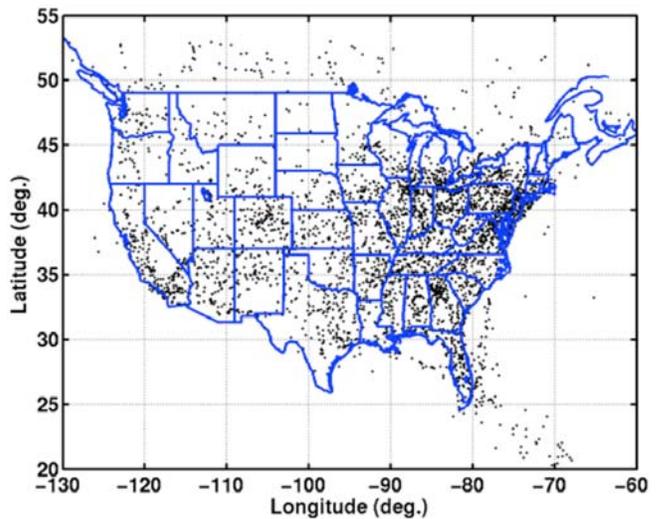
$$WITI(k) = \sum_{j=1}^m \sum_{i=1}^n T_{i,j}(k)W_{i,j}(k) \quad (11)$$



**Fig. 10. Regions of severe weather at 3:00 pm EST on July 16, 2005.**

where  $n$  is the number of rows and  $m$  is the number of columns in the weather grid.

Fig. 10 shows the locations where severe weather is indicated at 3:00 pm Eastern Standard Time (EST) on July 16, 2005. The weather grid consists of 1837 rows and 3661 columns, which are approximately one nautical mile wide. Fig. 11 shows the corresponding locations of aircraft in the weather grid based on historical demand. Finally, element-by-element multiplication of the two grids in Figs. 10 and 11 and summation in step 3 [see (11)] results in a WITI at 3:00 pm on



**Fig. 11. Regions of expected air traffic at 3:00 pm EST on July 16, 2005.**

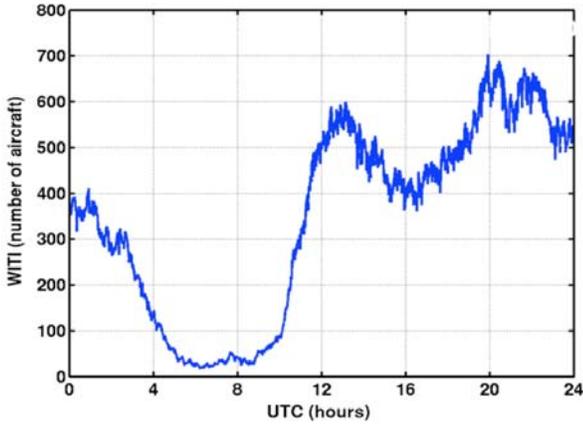


Fig. 12. WITI values as a function of time for July 16, 2005.

July 16, 2005. Fig. 12 shows the WITI values for that day as a function of time. Observe from (11) that the unit of WITI is number of aircraft, since  $W_{i,j}$  takes on values of one or zero. Thus, WITI is a weather weighted traffic count. Once WITI time histories, such as in Fig. 12, are computed, some features or measures of the them can be mathematically related to OPSNET delays, for example, using a least squares procedure. WITI features can then be used for estimating the delay for the days of interest based upon the established functional relation.

In [38], the sum of WITI values, which is equivalent to the mean (a feature of WITI time history), was related to average arrival delay. Including additional features can enrich the description of WITI. Given the WITI time history as in Fig. 12, [39] describes methods to compute statistical features, frequency-domain features, and time-domain features. In addition, two surface weather features, which are number of airports with low visibility and number of airports with high wind speed, are included in the estimation of NAS delay.

A functional relation between the en route WITI features, surface weather features, and the OPSNET delay can be established as follows. Let

$f_j(i)$  be the  $j^{th}$  feature for the  $i^{th}$  day,  $w_j$  be the  $j^{th}$  coefficient, and  $d(i)$  be the OPSNET delay on the  $i^{th}$  day. With these definitions, the functional relation is established as follows:

$$\underbrace{\begin{bmatrix} f_1(1) & f_2(1) & \cdots & f_r(1) \\ f_1(2) & f_2(2) & \cdots & f_r(2) \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \vdots & & & \vdots \\ f_1(s) & f_2(s) & \cdots & f_r(s) \end{bmatrix}}_F \underbrace{\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_r \\ w \end{bmatrix}}_w = \underbrace{\begin{bmatrix} d(1) \\ d(2) \\ \vdots \\ d(s) \end{bmatrix}}_d \quad (12)$$

where  $r$  is the number of features used and  $s$  is the number of days. Note that  $s \gg r$ . The coefficient vector  $w$  can now be determined using the least squares procedure as

$$w = (F^T F)^{-1} F^T d \quad (13)$$

with the  $F$  matrix and the  $d$  vector defined in (12). Then, the estimated delay on day  $q$ ,  $\hat{d}(q)$  is given by

$$\hat{d}(q) = \sum_{i=1}^r w_i f_i(q) \quad (14)$$

where  $w_i$  is from (13) and  $f_i(q)$  is  $i^{th}$  feature on day  $q$ .

Reference [40] suggested that the behavior of the NAS is highly nonlinear, and days with higher delays may behave differently from those with lower delays. Severe weather reduces the capacity of the NAS by reducing the available resources at the airport and in the airspace. In this respect, the NAS can be viewed as a queuing network, and as the demand for resources as a fraction of the NAS capacity (denoted as  $\gamma$ ) increases, the NAS delay (denoted as  $d$ ) exhibits the following relation:

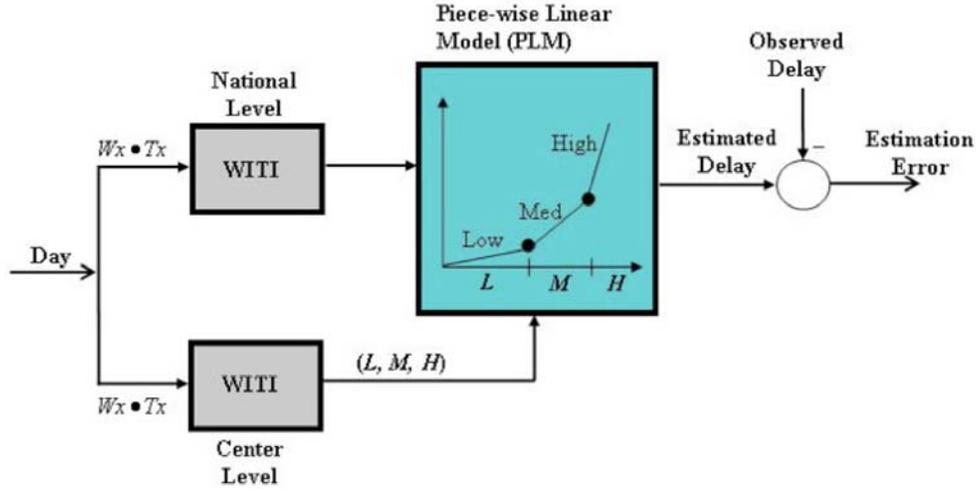


Fig. 13. Block diagram showing the WITI calculation, delay classification, and three-piece linear delay estimation model.

$$d \propto \frac{\gamma}{1-\gamma}, \quad (15)$$

where  $0 < \gamma < 1$ . It can be deduced from (15) that for low demand, the delay  $d$  is linearly proportional to  $\gamma$ , and for moderate demand,  $d$  is proportional to  $\gamma - \gamma^2$ . As demand reaches operational capacity, i.e., as  $\gamma$  approaches one,  $d$  increases exponentially. To better model the nonlinear nature of delay with demand, the linear model is extended in the next section.

Following [40], a piecewise linear model approximates the NAS behavior. Days are separated into three groups based on the magnitude of the observed delays during 2004–2006. The first group consists of days with low delays between 0 to 50,000 minutes; the second group of days is with medium delays between 50,000 to 100,000 minutes; and the third group is days with high delays of 100,000 minutes and above. The choice of these separation intervals, referred to as “knots,” is discussed in [39]. In each group, the NAS delay estimation model is developed by following (12)–(14). Recall in (12) the feature matrix  $F$ , as well as the OPSNET delay  $d$ , can be rearranged and partitioned into three submatrices/vectors corresponding to the three groups as follows:

$$F = \text{Diagonal}[F_1 \quad F_2 \quad F_3] \text{ and } d = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \quad (16)$$

where the subscripts 1, 2, and 3 correspond to each of the three groups of NAS delay days, respectively. Therefore, the coefficient vector  $w$  can also be partitioned into

$$w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \quad (17)$$

where  $w_i$  can be determined from the least squares procedure for  $i = 1, 2, 3$ , and is given by

$$w_i = (F_i^T F_i)^{-1} F_i^T d_i, \quad i = 1, 2, 3. \quad (18)$$

Thus, the estimated delay for each delay group is given by

$$\hat{d}_i = F_i w_i, \quad i = 1, 2, 3. \quad (19)$$

A detailed analysis of the three-piece linear models was performed in [40], and it was shown that the three-piece linear models predict the NAS delays better than a single linear model.

Fig. 13 is a block diagram showing the computation of the national WITI and center WITIs, the classification scheme, and the three-piece linear models [41]. It illustrates the integrated approach, which combines the delay classification with piecewise linear modeling to produce an improved delay estimation model.

## B. Passenger Delay Estimation Models

Flight delays measure the impact of TFM decisions on aircraft. The delays do not capture airline actions such as cancellations and diversions. They are not a true measure of the impact of delays to the end user, the passenger and business, dependent on the aviation system for transport. Some small delays in the system, leading to a situation where airlines are unable to maintain connectivity between different flights, may lead to large passenger delays. The passenger delay model [43] shows that flight delays alone can severely underestimate the delays faced by passengers with missed connections.

Inputs to the passenger delay model, which is also called the passenger delay calculator, include 1) the planned flight schedule, 2) the number of booked passengers and the fraction of passengers who book seats and show up at their flight legs, 3) the actual flight leg departure and arrival delays, and 4) the canceled flight legs [43].

Using these inputs, the passenger delay calculator 1) determines the set of disrupted itineraries and their respective passenger types; 2) places disrupted passengers in a recovery queue; 3) reschedules each passenger in the recovery queue on a recovery itinerary with the original airline if appropriate seating is available; 4) reschedules passengers not accommodated by the original airline with a recovery itinerary on an alternative airline; and 5) calculates the passenger delay statistics [43].

Some of the key findings from this study are that 1) “flight leg delays are not accurate surrogates of passenger delays for hub-and-spoke airlines”; 2) “connecting passengers are almost three times more likely to be disrupted than local passengers”; and 3) flight cancellation rates and the percentage of flights delayed by over 45 min

are more accurate indicators of passenger disruptions [43].

## C. Delay Propagation Models

Because the NAS is a complex and interconnected system, delays in one region routinely propagate throughout the domestic and international schedules of air carriers. Several research organizations have recently undertaken efforts to understand how delays propagate through this complex system [44]–[47]. For example, in [44] and [45], the use of Bayesian networks is proposed to investigate these effects, while in [46], simple statistical models are developed to gain insight into how delays propagate throughout the NAS.

Significant insights provided from these models include the observation that overscheduling at busy airports, such as New York’s LaGuardia Airport, creates situations in which there is no slack period during peak traffic hours to recover from early delays. This leads to a situation in which the only period during which the airport can recover from early delays is late at night after the scheduled demand reduces [46]. Additionally, these models indicate that the presence of one major carrier at an airport tends to make the airport operations more predictable [46].

## V. OPTIMIZATION

In this section, the TFM optimization problem is formulated as a general two-point boundary value problem (TPBVP), and subsequently dynamic programming approaches for solving this problem are discussed. Next, the state of the art in optimization approaches for scheduling flights to capacity constrained airports and en route regions of airspace is discussed, and an example in which flights are scheduled to Chicago O’Hare under both current and future traffic demand scenarios is presented. This section ends with a discussion of optimization approaches that account for uncertainties in forecasted air traffic demand and capacity.

### A. Two-point Boundary Value Problem

The TFM optimization problem can be formulated as a standard TPBVP with the systems dynamics represented by the equation

$$x(k+1) = f[x(k), u(k), w(k)] \quad (20)$$

where  $x(k)$  represents the state of the system at time  $k$ ,  $u(k)$  the control, and  $w(k)$  the disturbance in the system. Either the Lagrangian or the Eulerian models can represent the state of the system. If the state is represented by the position and velocity of each aircraft in the TFM problem, a 4-h planning period in the United States may involve about 15,000 aircraft, and the number of states in the system is about 90,000. The control variable  $u(k)$  is specified in terms of departure times of the aircraft from origin airport and the route followed by each aircraft from origin to destination.

The cost function can be specified in terms of the performance metrics described in Section IV. The number of stages  $N$  depends on the duration of the problem. Some commonly used metrics are total aggregate delay and deviations from schedule.

$$J = E \sum_{k=0}^N g[x(k), u(k), w(k)] \quad (21)$$

The major constraints in the TFM problem come from two sources: a) constraints on the number of arrivals and departures at an airport and b) the number of aircraft in a region of the airspace based on safety considerations. The maximum number of aircraft in a region, referred to as airspace capacity, depends on the ability of controllers and the automation tools to keep aircraft separated from each other. The capacity of the airspace is affected under severe weather conditions, as described in Section IV.

### B. Dynamic Programming

Dynamic Programming (DP) [49] provides an approach to the TPBVP described earlier involving optimization over a number of stages or time intervals. The decision at each stage has an

immediate cost but also affects the decisions possible at future stages. The cost of going from stage  $k$  to  $k+1$  is  $g[x(k), u(k), w(k)]$ . In selecting  $u(k)$ , one needs to think of not only  $u(k)$ , but also the desirability of state  $x(k+1)$  and the subsequent controls  $u(k+1), \dots, u(N-1)$ . The optimal cost-to-go  $I(x, k)$ , satisfies Bellman's equation

$$I(x, k) = \min_{u(k), \dots, u(N)} E \sum_{j=k}^N [g(x(j), u(j), w(j))] \quad (22)$$

$$= \min_{u(k), w(k)} E (g(x(k), u(k), w(k)) + I(x(k+1), k+1)) \quad (23)$$

At each  $k$ , it is optimal to use  $u(k)$ , which minimizes equation (22).

$$\hat{u}(x, k) = \arg \min_{u(k), w(k)} E [g(x(k), u(k), w(k)) + I(x(k+1), k+1)] \quad (24)$$

It is well known that DP for complex problems suffers from the "curse of dimensionality" and approximations to the cost function are needed to produce a suboptimal solution

$$\tilde{u}(x, k) = \arg \min_{u(k), w(k)} E [g(x(k), u(k), w(k)) + \tilde{I}(x(k+1), k+1)] \quad (25)$$

Most of the research in the application of dynamic programming looks for ways to approximate the cost-to-go function [50], [52]. This quick review of the DP methodology provides a background to look at the recent approaches used to solve different variations and approximations to the TFM optimization problem.

### C. Ground Holding Problem

Optimizing traffic flow management decisions has enticed researchers since late 1980s [52]. Most of the optimization methods in the literature have addressed the complexity and computational issues in the TFM optimization problem by considering special cases. They are formulated as linear and/or integer programming

problems, where the decision variables that are optimized are typically the departure (or ground) delay, rerouting, and airborne holding of individual flights or groups of flights.

Among the TFM problems, the ground holding problem (GHP) has been most thoroughly addressed. The GHP involves deciding on ground delay assignment to flights subject to the airport capacity constraints to minimize an objective function, which is typically the sum of ground and airborne delays weighted by their relative costs. Application of GHP lies in enhancing decision support systems for planning FAA-imposed GDPs.

Within the domain of GHP there are two subproblems: 1) the single airport ground holding problem (SAGHP) and 2) the multi-airport GHP (MAGHP). In the SAGHP, it is assumed that a single airport is constrained in the system due to arrival capacity-demand imbalance, while all other resources in the NAS are unconstrained. In the multi-airport case, as the name suggests, arrival and departure capacities of multiple airports are considered simultaneously while deciding on-the-ground delay assignment. The MAGHP, unlike the SAGHP, considers the network effect (or delay propagation) due to departure, and hence arrival delays.

In a deterministic setting, where airport capacities during future time-periods are assumed to be known with perfect information, the SAGHP can be formulated as a minimum cost network flow problem [53], [54]. The objective function minimizes either the total ground delay or some cost function of ground delays of individual flights. Since the cost of airborne delay of a flight is assumed to be more expensive than ground delay, in a deterministic setting any necessary amount of delay is absorbed in the ground, and hence the airborne delay cost component is not required in the objective function.

In the past, several researchers have focused on variants of the deterministic SAGHP. A set of banking constraints was added to the above formulation in [54]. Some major airlines commonly schedule banks of operations at their

hub airports, meaning a group of flights whose arrival (or departure) times fall within a specified time window. Such temporal grouping of flights facilitates the transfer of passengers, baggage, and airline crews in a hub-and-spoke system. Banking constraints proposed by [54] allocate ground delays to flights while keeping those in a bank temporally grouped.

In [55], an optimization model simultaneously assigned ground delays and optimally allocated an airport's runway capacity towards arrival and departure operations. The work in [56] also addressed a similar problem and provided a dynamic programming algorithm to enhance the efficiency in achieving optimal solutions. Subsequently, the authors of [57] extended their previous formulation to consider arrival and departure fix capacities at an airport along with that of its runways.

Typically, the FAA exempts certain flights, mostly based on the distance of their origin airport from the destination, from a GDP [58]. This mainly hedges against uncertainty in capacity forecasts for a couple of hours in advance. The motivation behind exempting long-haul flights is that if weather forecasts turn overly pessimistic and fair weather capacity eventuates, the delay of those flights will become unnecessary and unrecoverable. On one hand, this leads to increased efficiency in the case of imperfect forecasts; on the other hand, these flight exemptions cause systematic bias in slot allocation among airlines. In [59], a linear optimization model mitigated such an exemption bias in slot allocation. In essence, the objective function of their model minimizes the deviation between the assigned slot and the ideal slot of a flight. The ideal slot for a flight is determined by applying the ration-by-schedule (RBS) algorithm, which is based on the first-scheduled first-served principle, to all flights without any exemptions.

One of the major applications of the collaborative decision-making (CDM) paradigm has been in planning and implementing GDPs. Under CDM, the FAA allocates slots to airlines, who can then perform intra-airline cancellations

and slot substitutions to meet their operational goals such as on-time performance, speedy recovery from schedule disruption, etc. Typically, the FAA executes the RBS algorithm to allocate slots among airlines. Users have accepted RBS as an equitable allocation method. An extensive discussion on equity and fairness issues in slot allocation was provided in [60].

After a round of intraairline cancellations and substitutions, airlines notify the FAA about any changes in their schedule. Thereafter, a compression algorithm, which is essentially an interairline slot exchange mechanism, is executed to utilize slots vacated by flight cancellations. In recent years, a more dynamic form of interairline slot exchange, known as the Slot Credit Substitution, has been implemented in practice. In this model, airlines can submit conditional flight cancellation requests in exchange for delay reduction of another flight [61]. Optimization models in [62] suggested a more generalized form of interairline slot exchange during a GDP.

The deterministic MAGHP has received the attention of several researchers [63]–[65]. When large number of airports and flights are considered, this problem poses severe computational burden. A heuristic, based on a set of “priority” rules in assigning ground delays to flights, to solve the deterministic MAGHP was proposed in [66].

As in the SAGHP, there are several variants of the deterministic MAGHP. In [64], constraints were proposed that could capture the interdependence between arrival and departure capacities at airports. With this extension, it is possible to simultaneously determine the optimal ground holding strategy and the allocation of runway capacity between arrivals and departures. In all the above models for the MAGHP, a set of constraints, commonly known as flight connectivity constraints, is imposed between successive flights performed by one aircraft, i.e., single connections are accounted. An alternative formulation for the deterministic MAGHP that addresses the delay propagated by an aircraft onto several other flights, a situation that commonly

arises at hub airports where the arrival and departure times of several flights are interlinked, was proposed in [67].

#### D. Optimization Models for the Generalized ATFM Problem

Optimization models and algorithms that address en route capacity constraints treat the airspace system as a multiple origin-destination network on which traffic flow must be assigned. Deterministic optimization models addressing en route capacity constraints along with that of airports were formulated as a multicommodity network flow problem by [68], and more recently by [69]; the latter addresses routing as well as scheduling decisions. A deterministic model for deciding ground and airborne holding of individual flights under airport and airspace capacity constraints was formulated as a binary IP by [70]. More recently, [64] formulated a 0–1 IP to solve a similar problem. To date, their model performs best in terms of computation when applied to large-scale problems. Here we present the formulation in [64].

Let  $K$  denote a set of airports,  $F$  be the set of flights scheduled between those airports, and  $J$  denote the set of en route sectors. Let  $F'$  denote the set of pairs of flights that are continued, i.e.,  $F' = \{(f', f) : f' \text{ is continued by } f\}$ . Let the planning horizon be divided into  $T$  time intervals of equal duration. For a given flight  $f$ , let  $N_f$  denote the number of resources (i.e., sectors and airports), and  $P(f, i)$ ,  $1 \leq i \leq N_f$ , denote the  $i^{\text{th}}$  resource along flight  $f$ 's path. Note that  $P(f, 1)$  and  $P(f, N_f)$  represent the departure and arrival airports, respectively. Depending on the trajectory, each flight is required to spend a minimum number of time units,  $l_{fj}$  in a sector  $j$  that lies along its flight path. Let the capacity of resources during a time-interval  $t$  be denoted as follows:  $D_k(t)$  = departure capacity of airport  $k \in K$ ,  $A_k(t)$  = arrival capacity of  $k$ , and  $S_j(t)$  = sector capacity (i.e., number of aircraft allowed to be present in sector  $j \in J$ ). The flight-specific scheduled times

and delay costs are denoted as follows:  $d_f$ ,  $a_f$ , and  $s_f$  are the scheduled departure, arrival, and turnaround times respectively, while  $c_f^g$  and  $c_f^a$  denote the unit costs of delaying a flight in ground and in air.

The binary decision variables, which are nondecreasing, are defined as follows:

$$w_{ft}^j = \begin{cases} 1 & \text{if flight } f \text{ arrives at sector } j \text{ by time } t, \\ 0 & \text{otherwise.} \end{cases}$$

To reduce the size of the formulation, [64] proposed, for each flight, a feasible time-window when that flight can be present in a resource along its flight path. The feasible time-periods, for a flight  $f$  to be present in sector  $j$ , are represented by a set  $T_f^j$ ,  $j \in P(f, i), 1 \leq i \leq N_f$ . Based on the above decision variables, the total ground and airborne delays of a flight are given by the following expressions:

$$g_f = \sum_{t \in T_f^k, k=P(f,1)} t(w_{ft}^k - w_{f,t-1}^k) - d_f \quad (26)$$

$$r_f = \sum_{t \in T_f^k, k=P(f, N_f)} t(w_{ft}^k - w_{f,t-1}^k) - a_f - g_f \quad (27)$$

The objective function and the set of constraints are defined as follows:

$$\text{Min} \quad \sum_{f \in F} (c_f^g g_f + c_f^a r_f) \quad (28)$$

subject to:

$$\sum_{f: P(f,1)=k} (w_{ft}^k - w_{f,t-1}^k) \leq D_k(t) \quad \forall k \in K, t \in \{1, \dots, T\} \quad (29)$$

$$\sum_{f: P(f, N_f)=k} (w_{ft}^k - w_{f,t-1}^k) \leq A_k(t) \quad \forall k \in K, t \in \{1, \dots, T\} \quad (30)$$

$$\sum_{f: P(f,i)=j, P(f,i+1)=j'} (w_{ft}^j - w_{f,t-1}^{j'}) \leq S_j(t) \quad \forall j \in J, t \in \{1, \dots, T\} \quad (31)$$

$$w_{f,t+l_f}^{j'} - w_{ft}^j \leq 0 \quad \begin{cases} f \in F, t \in T_f^j, j = P(f, i), \\ j' = P(f, i+1), i < N_f \end{cases} \quad (32)$$

$$w_{ft}^k - w_{f',t-s_{f'}}^k \leq 0 \quad \begin{cases} (f', f) \in F', t \in T_{f'}^k, \\ k = P(f, 1) = P(f', N_{f'}) \end{cases} \quad (33)$$

$$w_{ft}^j - w_{f,t-1}^j \geq 0 \quad \begin{cases} \forall f \in F, j \in P(f, i), \\ 1 \leq i \leq N_f, t \in T_f^j \end{cases} \quad (34)$$

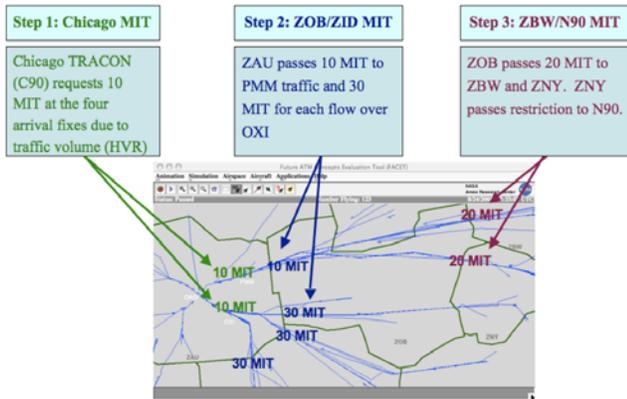
$$w_{ft}^j \in \{0, 1\} \quad (35)$$

The objective function minimizes the total cost of flight delays. The set of constraints are classified into two categories: capacity constraints [(29)–(31)] and connectivity constraints [(32)–(34)]. The capacity constraints ensure that the flow is bounded by the capacities of each resource in the system—airports and sectors. For example, constraint set (31) ensures that the total number of flights within a sector during any time interval does not exceed the sector capacity during that time period. Within the connectivity constraints, there are two subcategories: sector and flight connectivity. The sector connectivity constraints (32) ensure that each flight passes through the proper sequence of sectors in its route between origin and destination airports. The flight connectivity constraints (33) ensure that an aircraft must spend a minimum “turnaround” time at an airport before it can depart on its subsequent leg. Constraint set (34) ensures that the decision variables are nondecreasing, while (35) ensures they are binary.

There are several variants of the deterministic ATFM optimization problem. In [71] a binary IP was proposed for the TFM problem, which considers controller workload, airspace safety, and equity among airlines. Subsequently, in [72] and [73], the binary IP was extended to incorporate rerouting.

## E. An Example

This section presents an example of how the deterministic optimization methods for ATFM can be applied to improve decision-making in a



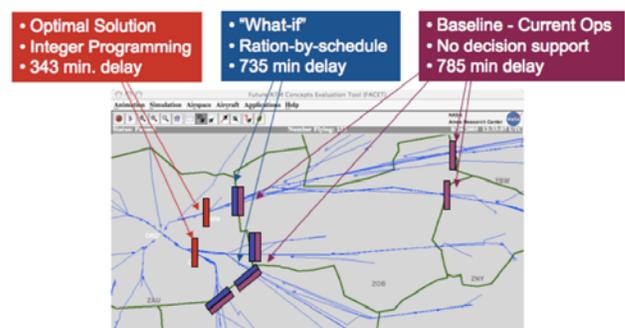
**Fig. 14. Historically validated miles-in-trail restrictions for Chicago arrivals.**

practical setting. Fig. 14 shows the flight tracks, on a weather-free day, for aircraft arriving at Chicago O’Hare (ORD) from various origin airports. For reference, FACET [19] generated and displayed the flight tracks from ETMS data. Due to the heavy volume of traffic scheduled for ORD, the Chicago TRACON (C90) implements 10 miles-in-trail (MIT) restrictions at the two east arrival fixes, Pullman (PMM) and Knox (OXI). Due to the limited airspace available in Chicago Center (ZAU) to meet these 10 MIT restrictions, the restrictions are subsequently passed back to the two centers adjacent to ZAU to the east, which are Cleveland Center (ZOB) and Indianapolis Center (ZID). The passed-back restrictions for the northernmost flow that arrives over PMM remains at 10 MIT; however, since three streams merge over the southern OXI fix, each stream under current operations typically receives 30 MIT. The rationale for applying 30 MIT is that if three traffic streams are spaced by 30 MIT restrictions, on average the combined stream will be spaced by 10 MIT, which of course is rarely the case. Finally, due to the limited available airspace in ZOB to properly space the northernmost flow that passes over PMM, the restrictions on this flow are typically passed back to New York Center (ZNY) and Boston Center (ZBW) as 20 MIT restrictions.

Fig. 15 illustrates three approaches to achieving 10 MIT for the ORD arrival flows that pass over the PMM and OXI fixes. The rightmost example in this figure represents the restrictions

imposed under current day operations, as previously discussed. The middle example represents a refinement over current day operations in which restrictions are not automatically passed back to ZNY and ZBW, while the leftmost example represents a scenario in which no restrictions are passed back. All 91 flights departing between 10:00 and 13:00 Coordinated Universal Time (UTC) that were destined for ORD, and crossed either PMM or OXI, were considered for this case study. The remainder of flights in ZNY, ZOB, ZBW, and ZAU Center were not controlled in this problem. The time-varying airport arrival, airport departure, and sector demand associated with these flights were calculated and subtracted from the nominal airport arrival rates, airport departure rates, and sector capacities to establish reduced capacities for these resources.

In terms of total delay, the approach used to manage ORD arrival flows under current day operations is the most overly restrictive and results in 785 min of delay. For this scenario, an RBS-based algorithm scheduled flights in order to meet the MIT restrictions at the locations of the six magenta rectangles in Fig. 15. When restrictions are only passed back to Chicago’s first-tier centers (ZOB and ZID), the results reduce to 735 min, which represents a 5% delay reduction. Finally, when a deterministic optimization model assigned departure delays to aircraft subject to the MIT restrictions placed at the airport arrival fixes, and en route sector capacity constraints were considered, the total delay reduced to 343 min



**Fig. 15. Three approaches to applying 10 miles-in-trail at the PMM and OXI arrival fixes.**

(i.e., savings of 56%).

The deterministic optimization model was applied to test the sensitivities of the results under varying demand and capacity, which are presented in Table 2. Clearly, if the traffic volume increases threefold, as is predicted in the Next Generation Air Transportation time-frame [4], without any increases in en route or surface capacity, the problem becomes infeasible. The infeasibility results from a constraint in the binary integer program model that limited the maximum ground delay of a flight to 200 min or less. In other words, demand increase in the NAS must occur in parallel with technological changes and capacity enhancements. From the table, it is evident that the ORD arrival fix capacities pose more severe constraints than en route sector capacities. For example, under the scenario in which en route sector capacities remain at the present-day level while the airport arrival fix capacities increases twofold, there is a feasible solution (total delay of approximately 13,000) compared to an infeasible solution when airport fix capacities remain at today's level. For a scenario with three times current traffic and two times ORD arrival fix capacity, increasing the en route sector capacity from two to three times does not reduce the total delay since the capacity is exceeded at the arrival fix. If capacities of all resources increase proportional to the forecasted demand growth, the average delay remains the same as the current level.

## F. Accommodating Uncertainty

Deterministic models for ground holding are applicable when airport capacities in the future hours are known in advance with perfect information. In reality, this is rarely the case. To date, efficient stochastic optimization methods have been developed to solve the single-airport GHP. A limited amount of literature exists on probabilistic decision-making models under en route congestion.

In [74], an IP formulation of the multiperiod, two-stage stochastic optimization model for the SAGHP was first proposed. In that model,

**Table 2 Sensitivity Analysis**

Traffic Demand	En route Sector Capacity	Chicago Arrival Fix Capacity	Delay (min.)	Average Delay (min.)
1X	1X	1X	343	4
3X	1X	1X	Infeasible	-
3X	2X	1X	18,606	73
3X	1X	2X	13,022	51
3X	2X	2X	4,718	19
3X	3X	2X	4,718	19
3X	3X	3X	1,008	4

uncertainty in airport capacity is represented by a finite set of scenarios, each of which represents a time-varying profile of the airport capacity that is likely to occur. The goal is to assign ground delays to flights in the face of uncertainty in airport capacity to minimize the total expected delay cost. More recently, [75] showed that for certain convex ground delay cost functions, the slot (i.e., ground delay) assignment from [74] model matches the first-scheduled first-served principle and, hence, is equitable.

A linear IP, the dual of which reduces to a minimum cost network flow problem to decide on the set of planned airport arrival rates (PAARs) given a set of possible scenarios of airport capacity, was proposed in [76]. An interesting property of the model in [76] is that the objective function is invariant to slot assignment and intraairline substitutions, which occur at the later stages. The output of this model (i.e., the set of PAAR) can be used to set GDP rates, which can then allocate slots to airlines. This makes the model in [76] the most relevant decision support tool for GDP planning under the CDM paradigm.

Both the models in [74] and [76] can be used to decide the optimum set of PAARs in face of uncertainty in airport capacity. Thereafter, a slot allocation algorithm, such as the RBS, must be applied to allocate arrival slots to individual airlines. Models that simultaneously decide on the PAAR and slot assignment to individual (or sometime groups) aircraft were proposed in [77] and [78]. These are based on multistage stochastic optimization based techniques [79], where ground-

holding strategies can be revised under updated forecasts. In addition to capacity scenarios, these models require as an input a decision tree, whose branching points and branches themselves reflect changing airport capacity profile.

A major challenge in using the scenario-based models in practice, as highlighted by [80], is the development of capacity scenarios and a decision tree. Statistical clustering technique to develop capacity scenarios for an airport based on historical data was proposed in [80]. More recently, [81] proposed a scenario-free sequential decisionmaking problem, based on dynamic programming techniques, for the stochastic SAGHP.

Stochastic optimization models that account for both airport and en route airspace constraints decide on rerouting along with ground holding aircraft. Like their deterministic counterparts, these problems are usually associated with a high degree of computational complexity. The problem of dynamically routing an aircraft under uncertainty in weather forecasts in a piece of airspace using Markov decision process was

addressed in [82]. An advanced methodology, which introduces robustness to the solutions when the components of the transition matrix are stochastic, was proposed in [83]. In [84] and [85], multistage stochastic optimization models for limited rerouting of flights arriving at an airport when weather blocks certain regions of the airspace in the vicinity were proposed. Their research showed significant promise in dynamically rerouting flights as opposed to static, predeparture route choice.

In [86], a statistical method predicted congestion at en route sectors using three main parameters: 1) look-ahead time, 2) predicted peak traffic count, and 3) traffic pattern at a given sector. These probabilistic congestion predictions were used in [87] to develop an incremental decision support system, based on Monte Carlo simulation, to control flight maneuvers such as ground delay and rerouting.

### G. Software

One of the primary challenges associated with applying the optimization approaches described in

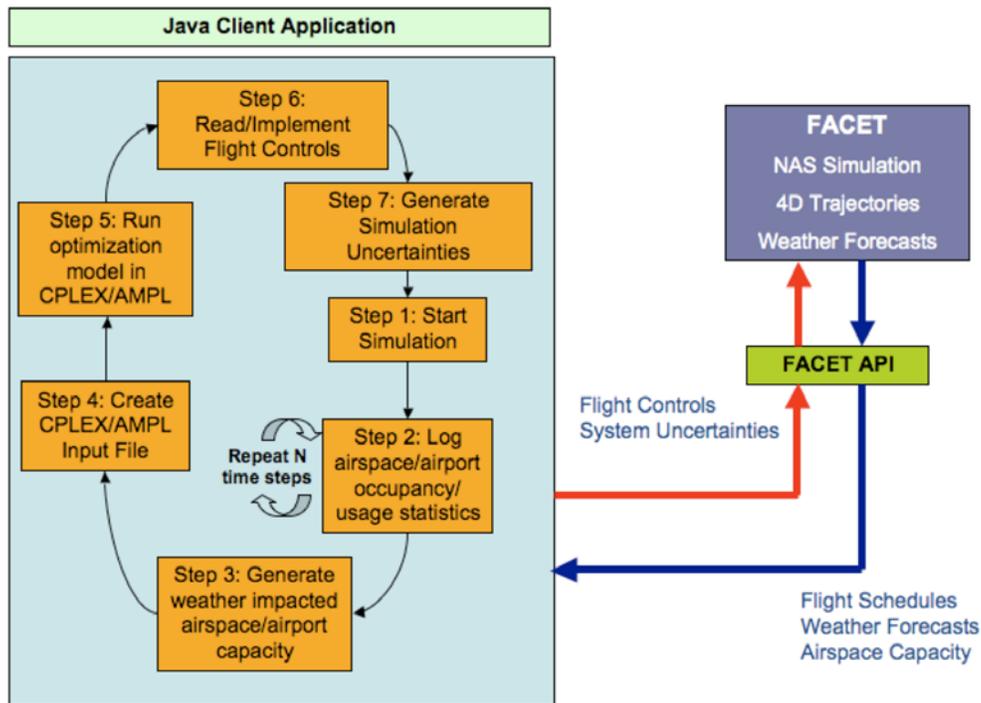


Fig. 16. Java-based client interaction with the FACET application programming interface.

the previous sections is supplying these models with operational flight, weather and airspace data, and assessing the performance of the flow control strategies developed by these models in realistic, large-scale NAS-wide modeling and simulation environments. This need for combining optimization and modeling approaches has been an area of recent interest [51]. To explore this issue, NASA has recently developed an application programming interface (API) for FACET [19] that leverages many of the core capabilities designed in the development of the Configurable Airspace Research and Analysis Tool-Scriptable (CARAT#) [88] system. More than 600 FACET methods are available through the API that cover a wide range of capabilities, ranging from starting a simulation to forecasting the future position of a set of scheduled flights. In addition to methods for accessing information about the state of aircraft and the airspace, methods also exist for rerouting flights, departure delaying flights, and imposing airborne flight delays.

A graphical depiction of a Java-based client that utilizes the FACET API, and was used in generating the results presented in Section V-E, is illustrated in Fig. 16. Starting at the box labeled “Step 1: Start Simulation,” the FACET application is instantiated and an N minute simulation is started with a user-specified input file containing a list of active and scheduled flights. For the purpose of this study, the list of scheduled and active flights were obtained from a historical ETMS [6] data file, and the simulation planning horizon was set to 3 h. After starting the simulation, the software underlying the box labeled “Step 2: Log airspace/airport occupancy/usage statistics” was used to create a Java hash table of “Aircraft” objects that was used to record the entry time and usage time for all airports and sectors along the flight path of each scheduled and active flight in the simulation. Following the N minute simulation, the nominal airport and airspace capacities were systematically reduced in the step labeled “Step 3: Generate weather impacted airspace/airport capacity” to

simulate the impact of constraints, such as weather-induced en route capacity constraints.

In the box labeled, “Step 4: Create CPLEX/AMPL Input File,” the forecasted aircraft and airspace demand data is formatted using A Modeling Language for Mathematical Programming (AMPL) [89] to provide the inputs to the previously defined binary integer programming model. After casting the binary integer-programming model presented in Section V-D in the AMPL format, both the AMPL formatted input file and model file are passed into ILOG’s AMPL/CPLEX optimization software to assign the optimal departure delays to the set of flights included in this study. This optimization occurs in the section of the code labeled “Step 5: Run optimization model in CPLEX/ AMPL” in Fig. 2, and version 10.0 of AMPL/CPLEX was installed on a Redhat Linux based-laptop computer with a 3.0 GHz Pentium 4 processor and 1 GB of RAM to generate the results appearing in Section V-E.

Finally, the two sections of the Java client that are labeled “Step 6: Read/Implement Flight Controls” and “Step 7: Generate/Introduce Simulation Uncertainties” are used to implement the optimal airborne and departure delay controls in FACET, and to explore the impact of demand and capacity uncertainties on the model’s solutions.

## VI. CONCLUSION/SUMMARY

This paper provides an overview of how TFM decisions are made today and challenges facing the system in the future, and reviews the modeling, simulation, and optimization approaches for TFM facilitating system-wide modeling, performance assessments, and decision-making.

In the area of system-wide modeling, recent advances in the development of aggregate, or Eulerian, traffic flow models were discussed. Three models recently appearing in the literature were highlighted. The first model uses flow relations between adjacent centers to create a time-varying linear dynamic system model. The

second model spatially aggregated the flows in a network of interconnected, two-dimensional control volumes, and leveraged prior research from the area of highway traffic modeling. In the third approach, a fully continuous model based on a modified Lighthill–Whitham–Richards [30] partial-differential equation was proposed.

The second major area covered in this paper was recent approaches designed to assess the performance of the national airspace system. Models based on the Weather Impacted Traffic Index concept, a passenger delay estimation model, and models that explore the propagation of delays throughout the NAS were examined.

Finally, in the area of optimization, the basic TFM problem was cast as a two-point boundary value problem, and a dynamic programming approach to solving the problem was discussed. Subsequently, the state of the art in optimization

approaches for scheduling flights under both deterministic and stochastic airport and airspace constraint scenarios was discussed. An example in which flights are scheduled to Chicago O’Hare under both current and future traffic demand scenarios was presented to illustrate the use of these models.

Ultimately, the success of the TFM modeling and optimization approaches will be measured by their ability to improve the efficiency of TFM decisions in the current and future systems. The example presented in this paper shows the potential savings from an optimization approach. The actual savings will depend on policies and improvements to the NAS that will enable TFM decisions to be communicated to a large group of decision-makers and executed in a cooperative manner. ■

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