
The N/Rev Phenomenon in Simulating a Blade-Element Rotor System

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ABSTRACT

When a simulation model produces frequencies that are beyond the bandwidth of a discrete implementation, anomalous frequencies appear within the bandwidth. Such is the case with Sikorsky Aircraft's sophisticated blade-element models of rotor systems, which are used by NASA in the real-time, man-in-the-loop simulation environment. Steady-state, high-frequency harmonics generated by these models, whether aliased or not, obscure piloted-helicopter simulation responses. Since these harmonics are attenuated in actual rotorcraft (e.g., because of structural damping), a faithful environmental representation for handling-qualities purposes may be created from the original model by using certain filtering techniques, as outlined here. These include harmonic considerations, conventional filtering, and decontamination. The process of decontamination is of special interest because frequencies of importance to simulation operation are not attenuated, whereas superimposed aliased harmonics are.

INTRODUCTION

Sikorsky Aircraft Company has provided Ames Research Center with real-time simulation models of both the RSRA (1) and Black Hawk (2) rotorcraft. The rotor systems of these models are described in terms of rotating blade-element techniques, which are preferred, for engineering reasons, to other methods that are less exacting computationally. These reasons include the capacity to model diverse rotor types, and the fact that these blade-element techniques generally produce rotorcraft simulations that are applicable over extensive operational regions. The N/rev frequency contribution is produced, where N is the number of blades, as are the associated harmonics of this fundamental frequency. These frequencies, which have significant power, are a consequence of real, nonlinear physical relationships. However, they can be destructive to discrete, real-time simulations (3,4). Their appearance as contributions to the vehicle's dynamics can cause problems ranging from fictional tracking tasks to deterioration of simulator hardware.

The Black Hawk simulation model is here used to illustrate the N/rev phenomenon, as constrained by realistic computational bandwidths, and to give insight into the computer science techniques that may be used to suppress undesirable N/rev multiples.

FREQUENCY CONTENT

Figure 1 displays the Black Hawk simulation's angular acceleration history during a typical dynamic check; the cycle time is 20 msec and the vehicle velocity is 100 knots. Three stages are shown in this figure. The first stage is the original model (Fig. 1a),

in which both real and aliased frequency content obscure the vehicle response. The last stage (Fig. 1c), in which steady-state frequencies have been eliminated, will be our final result.

The transition to the second stage (Fig. 1b) appears to be quite dramatic, but it is accomplished by two rather straightforward techniques, outlined as follows.

Purging Inertial Shears

Normally, some inertia terms in a rotor model sum to zero over the blade index, but in the RSRA model those terms were retained by Sikorsky in order to permit the evaluation of out-of-balance loads following blade severance. This extensive inertial description was carried over to the Black Hawk model. At Ames Research Center, terms contributing high-frequency content in the inertial description were examined, and by using Fourier techniques their low-frequency contributions to vehicle performance were extracted. An alternative formulation was then developed, installed in the model as an option, and investigated using time-scale techniques. Over the entire flight profile, neither trim points nor dynamic checks were influenced, except that some very-high-frequency content was eliminated.

N/Rev Notch Filter

As will be discussed, the dominant frequency output of a rotor model appears at a frequency given by N (the number of blades) times the rotor rpm. For the Black Hawk helicopter this frequency is approximately 17.2 Hz, which is well beyond simulator and pilot responses, and hence immaterial to handling-qualities investigations. A discretely designed notch filter was therefore used on the total rotor system's force and moment outputs, where the notch frequency was N/rev.

FREQUENCY ISOLATION

By purging the inertial shears of high-frequency terms and applying a nonrecursive notch filter to the total rotor-system outputs, the traces of Fig. 1b are created. However, the yaw axis remains contaminated by a 1.5-Hz limit cycle, and this persistent frequency may be used to illustrate the N/rev phenomenon in simulating a blade-element rotor system. The 1.5-Hz signal is actually a harmonic of N/rev that may be eliminated from the low-frequency (operational) region by the application of a "decontamination algorithm." This algorithm produces the traces shown in Fig. 1c. Frequencies of interest remain intact, but the rotor-generated harmonic at 1.5 Hz is purged. Thus, the low-frequency region is clear of all but intentional vehicle and pilot inputs.

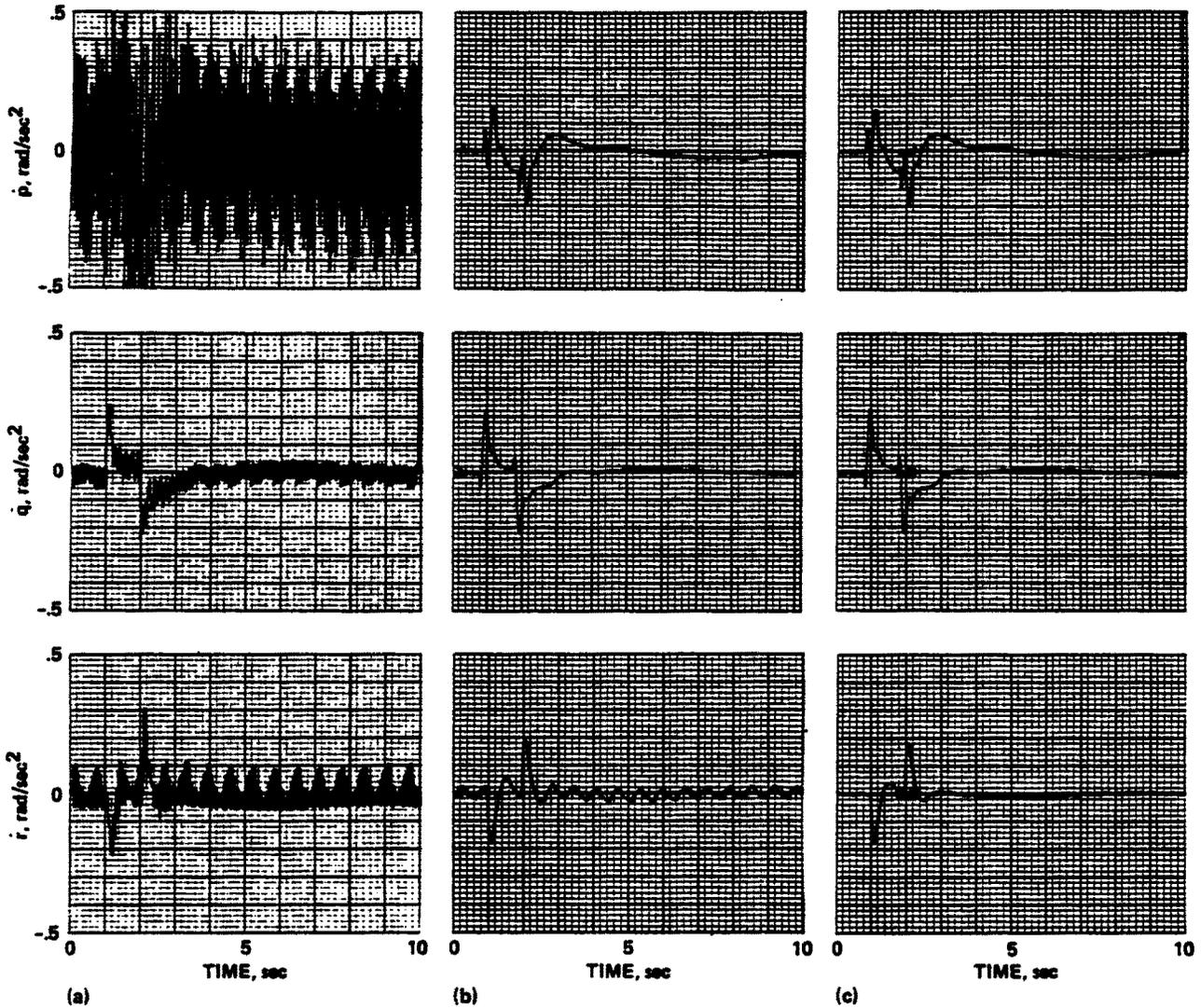


Figure 1 Angular Accelerations: Vehicle Velocity $V = 100$ knots; Cycle Time $T = 20$ msec.
 (a) Original Model; (b) After Simplifying; (c) Final Model.

HARMONIC FREQUENCIES

Under quiescent flight conditions, power is generated by the individual blades in a rotor system at multiples of the rpm as a consequence of their periodic physics. As a consequence of the orthogonality of sines applied to these harmonic signals, the total rotor system displays this power at multiples of N/rev , and eliminates other rpm multiples. The mathematical basis for the seemingly large magnitudes associated with these high frequencies will be discussed below, but for the present let us assume that a rotor system generates appreciable power at N/rev harmonics under steady-state conditions.

N/rev harmonics may easily extend beyond the computational bandwidth when realistic cycle times are used. This is examined in Fig. 2, where the first four multiples of N/rev are mapped as functions of cycle time. Any aliased frequency in the model may be computed from its origin, using the given aliasing equation, where the half-bracket operation is the "least integer" or "floor" operator. In particular,

note that the $3N/\text{rev}$ frequency folds to 1.5 Hz at a cycle time of 20 msec.

A feasible cycle time is closely associated with the convergence features of these harmonics, and variable rpm is also a consideration. If power beyond $4N/\text{rev}$ was known to be absent, for instance, a cycle time of 24 msec would most probably be superior to a cycle time of 20 msec, within limitations. If rpm variations remained very small, for instance, then the location of aliased harmonics would remain well above the bandwidth of simulation hardware and pilot responses, although vehicle responses would still be obscure in time-series data.

As shown in Fig. 3, the use of an unrealistically small cycle time (5 msec) reveals the spectral power. The peaks in this figure are designated according to their harmonic origins up to the astonishing value of $12N/\text{rev}$, or 206 Hz. The frequency locations conform to the aliasing equation.

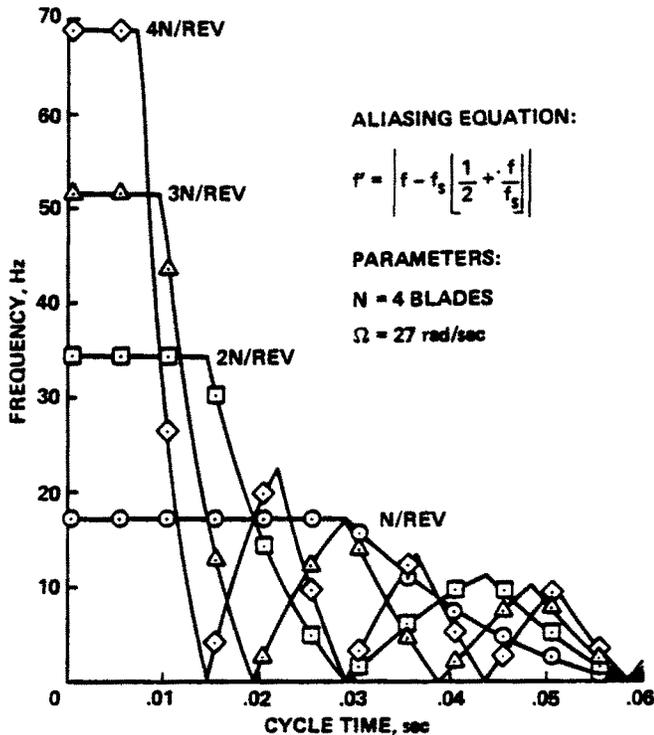


Figure 2 Black Hawk N/rev Aliasing

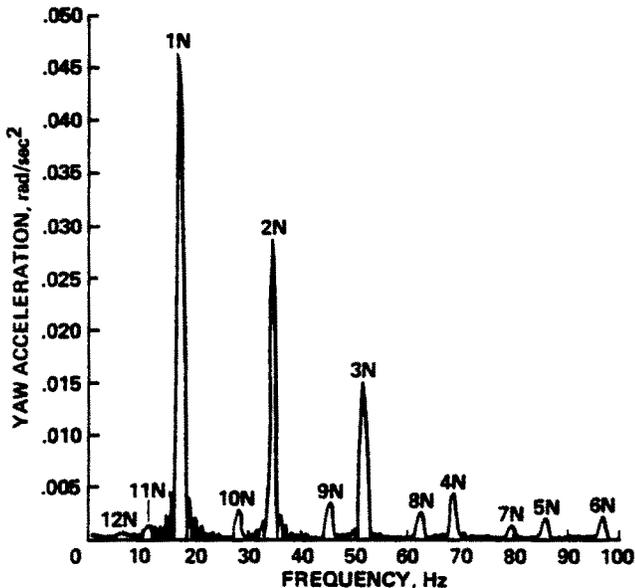


Figure 3 Black Hawk Rotor System Yaw Spectra:
V = 100 knots, T = 5 msec

By using much more realistic cycle times, such as the 20- and 30-msec times shown in Fig. 4, the harmonic power appears within the applicable Nyquist frequency, and contaminates the low-frequency region. For the 20-msec case, the 3N/rev harmonic appears at the 1.5-Hz position. At a cycle time of 30 msec, the low-frequency contamination is considerable because its major source is the very energetic 2N/rev signal.

The 20-msec case (Fig. 4a) is the yaw acceleration spectrum of the original model, where the time-history was given in Fig. 1a. In transition to the model stage that contains the "simplifying operations," only the 1.5-Hz signal remains pertinent to this discussion; it alone survives the purging of inertial terms and N/rev output filtering. This signal will be shown to have a harmonic origin that is over 2 times the Nyquist frequency of 25 Hz (20-msec cycle time).

HARMONIC ORIGINS

By investigating the spectrum of various system variables within the mathematical model of the rotor system, the lag-damper description at the blade level of computation was isolated as the major source of the 3N/rev signal. The lag-damper nonlinearity consists of a table lookup function which models an odd-symmetric saturation element with a small linear region. The axial rate of the lag-damper arm for each blade is the input to this nonlinearity; as shown in Fig. 5, where rpm multiples are indicated, this rate consists of rapidly decreasing harmonics of the rpm. An m th harmonic (of the rpm) component in the axial rate may be represented by

$$f_{m,n,k} = r_m \sin(m\psi_{n,k}) \quad (1)$$

where the azimuth angle of a particular blade is

$$\psi_{n,k} = k\Omega T + 2\pi(n-1)/N \quad (n = 1, 2, \dots, N) \quad (2)$$

$$(k = 1, 2, \dots)$$

Mathematically, the lag damper is an odd function, so only odd harmonics of the input spectrum are generated. An infinite set of odd harmonics of the input [Eq. (1)] is produced:

$$g_{m,n,k} = \sum_{i>0} R_{m,2i-1} \sin[(2i-1)m\psi_{n,k}] \quad (3)$$

This process is shown in Fig. 5b and reiterated in Fig. 6a, or after the conversion from force to moment space. These lag-damper moments propagate throughout the rotor model, providing damping to the individual blade's differential equations describing lagging and flapping motion. However, of particular concern to us here is the direct summation of these damper signals over the blade index, in contribution to the total rotor system's torque output. The selective spectral windows then become clear from the orthogonality of the system, that is,

$$h_{m,k} = \sum_{n=1}^N g_{m,n,k}$$

$$= \begin{cases} \sum_{i>0}^N R_{m,2i-1} \sin[(2i-1)mk\Omega T] & (2i-1)m/N \text{ integer} \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

Integer multiples of the rpm that are not also multiples of the number of blades do not survive this summation process. Also, since N is even, only odd harmonics of the N/rev signals from the lag damper are transmitted, with an amplitude gain given by N .

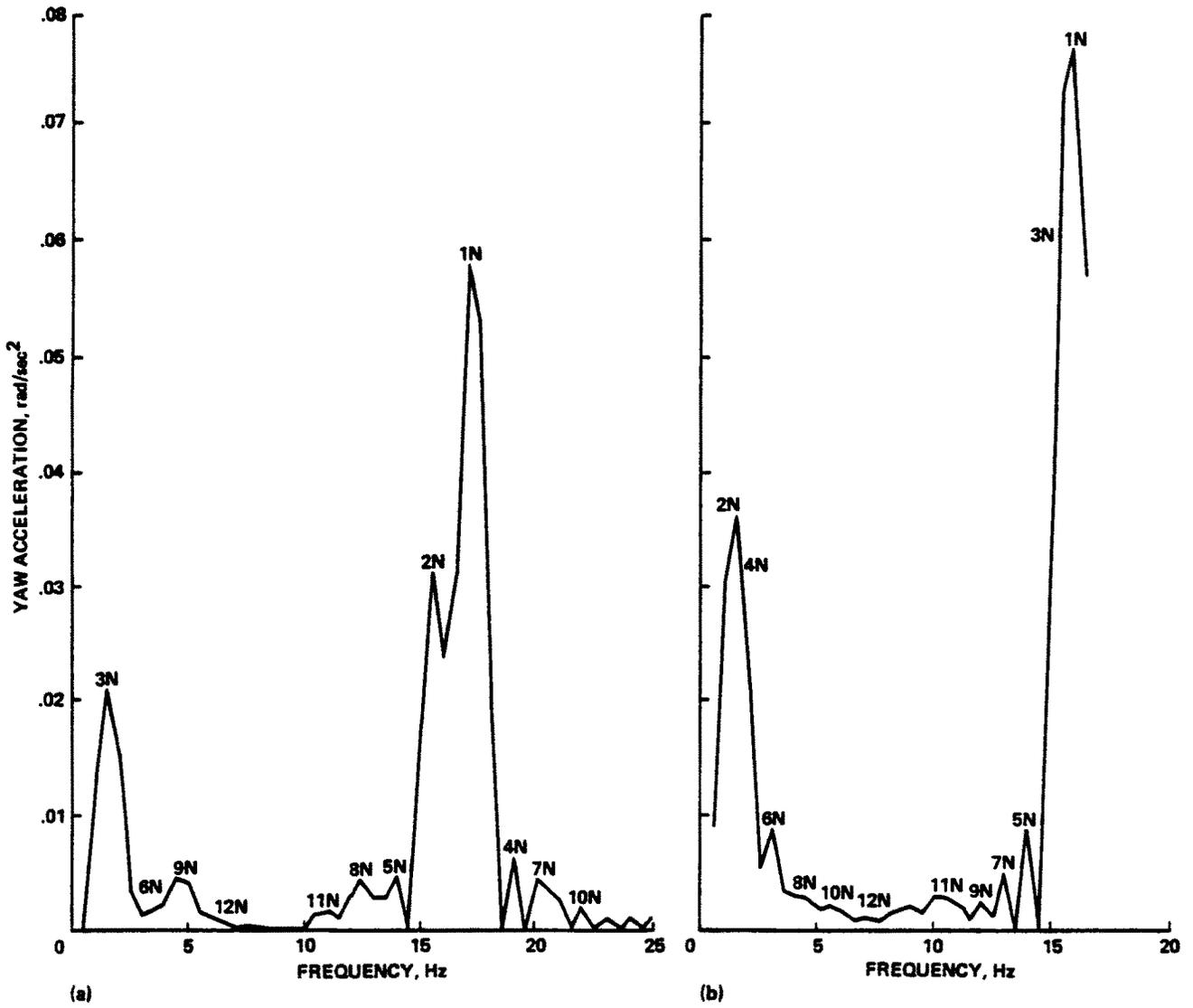


Figure 4 Black Hawk Yaw Spectra at Realistic Cycle Times. (a) $T = 20$ msec; (b) $T = 30$ msec.

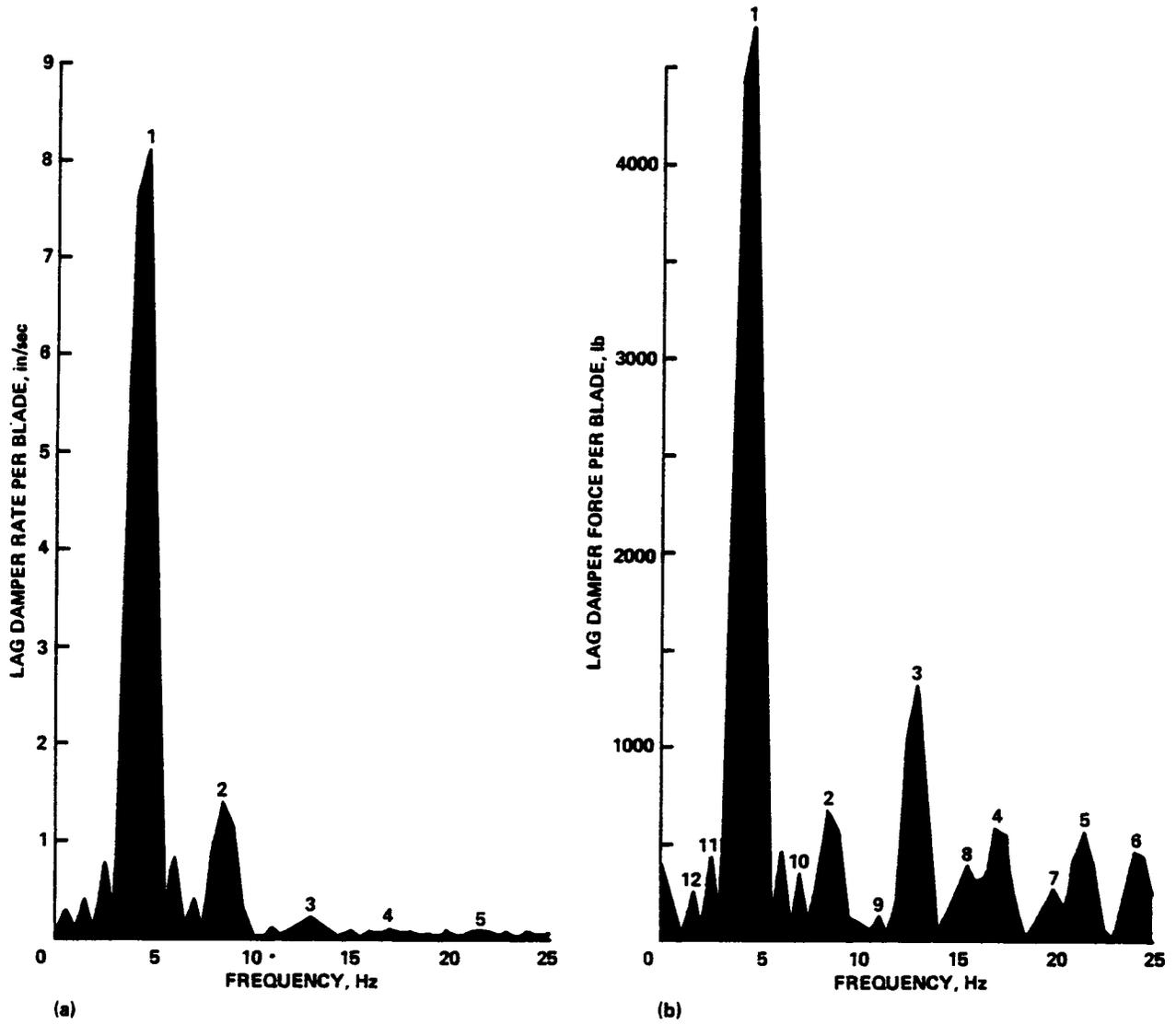


Figure 5 Saturation Element Input/Output Spectra: $V = 100$ knots; $T = 20$ msec.
 (a) Input Spectrum; (b) Output Spectrum.

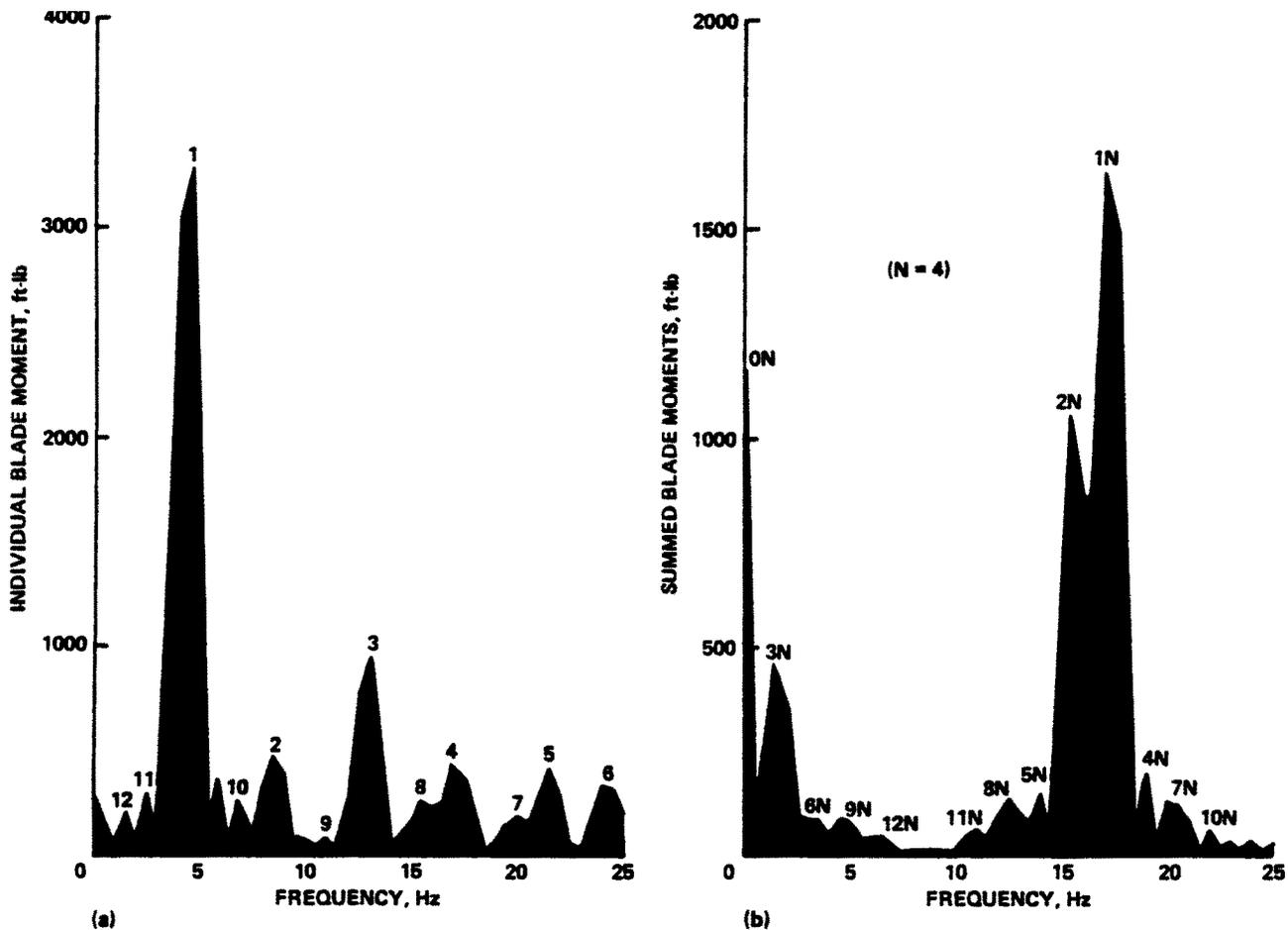


Figure 6 Lag-Damper Moment Spectra: $V = 100$ knots, $T = 20$ msec.
 (a) Individual Moments; (b) Summed Moments.

After summation over the blade index (Fig. 6b), most of the yaw output spectrum for the original rotorcraft model may be identified; 1/rev, 2/rev, and 3/rev have vanished under summation. Only multiples of N /rev have survived, and this feature permits the identification of origins on the left with consequences on the right. Both 4/rev and 8/rev on the left are gained by a factor of 4 (number of blades) as they appear in Fig. 6b. The 12/rev (or $3N$ /rev) signal is the first surviving odd multiple of N /rev, which would appear at about 51.5 Hz without aliasing. Its magnitude of approximately one-third of the N /rev magnitude compares favorably to a Coulomb-friction model for the lag damper.

HARMONIC CONSEQUENCES

Blade frequencies that are multiples of both the rpm and the number of blades have unusual significance in creating rotor-system outputs, although their usefulness in real-time simulation is questionable. The 4/rev blade-level signal produces the fundamental N /rev system output, along with its harmonic $3N$ /rev, which cannot be properly accommodated except with cycle times of less than 10 msec. With a 20-msec cycle time, this energetic harmonic folds to the annoying 1.5-Hz position, in conformance with the Shannon sampling theorem, and creates pilot workload where none should exist.

The 8/rev signal also survives the summation process, and is aliased to the approximation position of the N /rev signal, as shown in Fig. 6. This superposition assures that even the magnitude of N /rev is not faithful. However, the superposition also accrues an advantage. The $2N$ /rev signal is conveniently suppressed when the total rotor-system outputs are operated upon by the N /rev notch filter. Thus, using the 20-msec cycle time, only the $3N$ /rev signal constitutes significant contamination in the bandwidth.

NESTED SUBSYSTEM

In real-time simulation, a cycle time of a given value requires an actual computer workload of something slightly less than that value. Hence, the option of decreasing the cycle time by any significant amount is not available. However, if just some minor subsystem of the model required solution at a higher rate, then perhaps this could be done without materially influencing the computer workload. This technique may be called "quasi-cycles," and it has been used with good results when the numerical stability of some partially closed-loop subsystem required it. The quasi-cycle technique decreases computational degradation in the subsystem, so it more accurately creates the high-frequency content which here manifests itself as a low-frequency problem. The technique does not address the aliasing problem, because the final outputs of the

interpolated system must be decimated in order to communicate with the rest of the simulation model, and this operation aliases the harmonics.

When the causal nonlinearity can be identified and isolated, as in the Black Hawk model, the decontamination algorithm makes use of the quasi-cycle technique. This is shown in Fig. 7, where the notch frequency is 3 times N/rev, and the effective quasi-cycle Nyquist frequency has been increased to the inverse of the original cycle time.

The nested subsystem requires interpolation of inputs, which in this case doubles the Nyquist frequency. The nonlinear operation, here performed at the expanded rate, creates the harmonics. Since the nonlinearity is an odd function, the first harmonic of N/rev is 3N/rev, with a relative magnitude of approximately one-third. By expanding the Nyquist frequency to include this harmonic without serious aliasing problems, the harmonic may be operated upon by a conventional filter. This filter is designed to eliminate the power in the 3N/rev harmonic so that when decimation to the original Nyquist frequency occurs, the operational region is decontaminated.

THE FILTER

The discretely designed filter used in the decontamination algorithm requires that the selected notch frequency be located within the interval between one-half and three-halves of the Nyquist frequency of the quasi-cycle. For our particular example, the quasi-cycle has the expanded Nyquist frequency of 50 Hz, or twice the original, and the 3N/rev frequency is about 51.5 Hz. Hence, the interval criterion is satisfied. Using three sequential inputs at the expanded computation rate, given by $g(k-1)$, $g(k-1/2)$, and $g(k)$, the filter output relationship is

$$h(k) = \frac{\frac{1}{2}g(k) + \frac{1}{2}g(k-1) - \cos\left(\frac{1}{2}HT\right)g\left(k-\frac{1}{2}\right)}{1 - \cos\left(\frac{1}{2}HT\right)} \quad (5)$$

where H is the notch frequency in rad/sec, and T is the original cycle time. The filter response to a sine wave of frequency w is thus,

$$g(k) = \frac{\cos\left(\frac{1}{2}wT\right) - \cos\left(\frac{1}{2}HT\right)}{1 - \cos\left(\frac{1}{2}HT\right)} \sin\left[\left(k - \frac{1}{2}\right)wT\right] \quad (6)$$

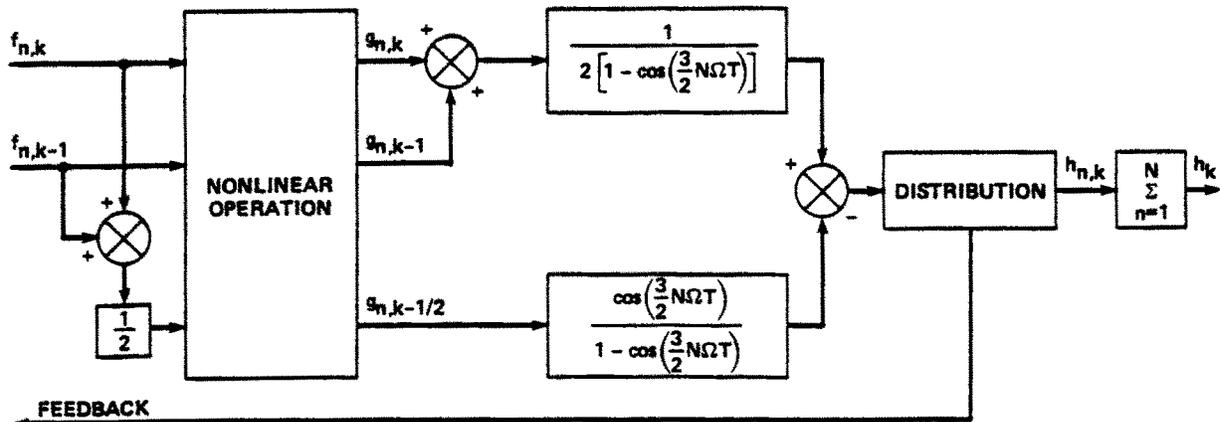


Figure 7 Implementation of the Decontamination Algorithm

The gain of this filter is shown in Fig. 8, where multiples of the rpm are indicated. In the operational region, limited by perhaps 5 Hz in our simulation environment, the attenuation is negligible; the linear phase in the operational region is also negligible; and at the N/rev harmonic locations it is immaterial.

The decimation process is indicated by the dashed line in Fig. 8; rpm multiples higher than five are aliased to positions below the Nyquist frequency of 25 Hz, but the filter provides attenuation prior to aliasing. In particular, the operational region is cleared of rotor-generated power.

As shown in Fig. 9, rotor-generated power is flight-regime dependent. Transient behavior excites these harmonics to a greater extent than the quiescent conditions studied here, so that notch filtering is an appropriate technique for suppressing power in the N/rev spectral windows.

CONCLUSIONS

Because of bandwidth limitations in real-time simulation, high-frequency content is not generally considered, although the modeled physical entity may itself suppress these frequencies by some phenomenon such as structural damping. The elimination of certain high-frequency terms in the inertial description and the implementation of an N/rev notch filter suppresses many of these frequencies, but these procedures cannot handle frequencies that are aliased into the operational region. The decontamination algorithm provides a technique for eliminating the aliased frequencies, while preserving input-output relationships that are considerably more important to man-in-the-loop operations.

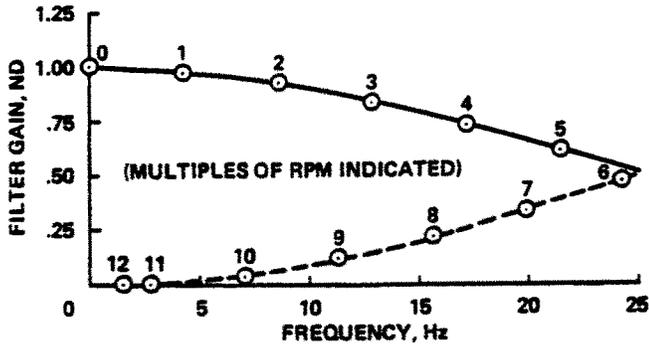


Figure 8 Filter Gain with Decimation

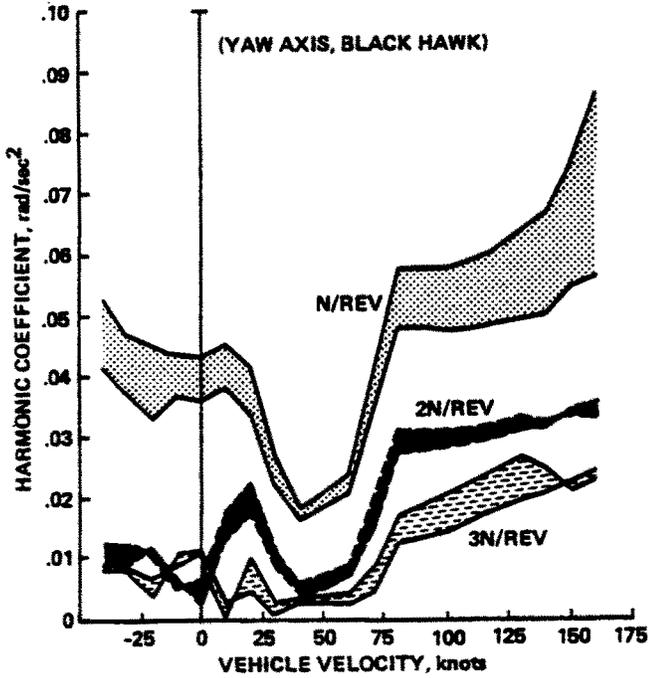


Figure 9 Spectral Dependence with Flight Regime: Black Hawk, Yaw Axis

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