An Aggregate Sector Flow Model for Air Traffic Demand Forecasting

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Sector capacity, number of aircraft permitted in a region of the airspace referred to as a sector, is used to limit air traffic to an amount that can be safely handled by a human controller. The traditional approach for predicting traffic demand in a sector involves the simulation of trajectories of individual aircraft. The demand provided by this approach is inaccurate for hourly time horizons. This paper explores the use of linear time varying aggregate models constructed using historical data for predicting sector demand. Since air traffic varies with the seasons, day of the week and weather, multiple aggregate models can be built to represent different situations. The paper evaluates the accuracy of using a single aggregate model and compares it with the improvements that can be achieved by using several aggregate models and selecting the best model based on hypothesis testing. The paper also presents the results of using a combination of probability-weighted predictions from several aggregate models. Evaluation of the prediction errors for all the high-altitude sectors in Indianapolis Center for a month shows that the multiple aggregation models result in smaller errors; errors vary with the sector and do not vary significantly with the prediction interval.

1. Introduction

The demand for air transportation in the United States is expected to grow significantly over the next twenty years. New modeling and optimization techniques are needed for predicting and resolving demand-capacity imbalances in the airspace. Sector capacity, the number of aircraft permitted in a region of the airspace referred to as a sector, is used to limit air traffic to an amount that can be safely handled by a human controller. The traditional approach for predicting sector demand consists of propagating the current location of the aircraft forward in time using an aircraft performance model, flight-plan information and Traffic Flow Management (TFM) restrictions. The predicted locations are then used to determine the number of aircraft in sectors at future time instants. Thus the traffic demand is based on the known TFM plans at the beginning of the prediction interval. The approach is illustrated in Fig. 1. This trajectory-based approach is used in the Federal Aviation Administration’s (FAA) Enhanced Traffic Management System (ETMS)\(^1\), the Center TRACON Automation System (CTAS)\(^2\) and the Future ATM Concepts Evaluation Tool (FACET).\(^3\)

The trajectory-based models predict the traffic demand adequately for short durations of up to 20 minutes. The accuracy of predictions decreases with the increasing prediction interval. A recent study\(^4\) shows that the standard deviation of the sector count for a two-hour prediction varies from 3 to 5 aircraft. The sector demand is the output of a complex system subject to random errors and systematic feedback errors resulting from the difference between actual and proposed plans. There is a wide variation between the actual and proposed departure times of aircraft.\(^5\) The flight plans of both the aircraft in the air and on the ground may be modified both by the air traffic service provider and the airlines in response to congestion and bad weather. Efforts have been made in the past few years to improve sector demand predictions. Gilbo\(^6\) proposed a smoothing approach to reduce the inherent errors in the traffic demand prediction in ETMS and validated the results by considering quiet days when the results are not impacted by TFM actions such as ground delay program, re-routing or miles-in-trail.

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In recent studies\textsuperscript{6-8}, the dynamics of air traffic flow is modeled by Linear Dynamic System Models (LDSM). The dynamics of air traffic in the U.S. National Airspace System (NAS) is modeled by using flow relationship between adjacent centers in Ref. 7. Based on the historical observations, the LDSM in Ref. 7 is built by tracking the number of aircraft entering and leaving between each adjacent center, and the numbers of aircraft landing and taking off within a center over a defined time period. The time-invariant LDSM proposed in Ref. 7 is extended to a time-varying one in Ref. 8 where multiple state transition matrices are used to cover the entire prediction period. The resulting model is then applied to predict aircraft counts in the 23 airspace regions which correspond to the 20 centers in the continental United States, one each for Hawaii and Alaska, and one for the international airspace.

The objective of this study is to extend application of the time-varying LDSM\textsuperscript{8} to develop strategic traffic flow models for NAS that can forecast demand in sectors in the presence of uncertainties such as weather conditions. Several earlier studies\textsuperscript{9,10} show that days of the year can be classified based on the impact of weather, traffic demand and seasons. Therefore, multiple LDSM(s) are built such that each model corresponds to a weather, traffic and season classification. Then, hypothesis testing techniques are applied to select the reference model according to the characteristics of real-time traffic flow. Since real-time air traffic varies over time, a decision rule derived from a dynamic programming approach\textsuperscript{11} can be used to determine the optimal time of switching between best matching models. In this study, a combination of reference models is used instead of switching between best reference models. Air traffic demand estimated from each reference model is weighted by its own likeliness of matching the real-time air traffic. The predictions formed by combining these probability-weighted estimates is used for predicting air traffic during the next time period of about two hours. The method was tested by building 31 models representing August 2007 traffic in the sectors belonging to Indianapolis Center. The multiple models and a combination model were used to predict traffic counts and the errors between the model demand and actual demand for the month of September 2007. The analysis shows that the combined model has smaller errors; errors vary with the sector and do not vary significantly with the prediction interval.

The paper is organized as follows: Section II provides the development of an aggregate sector model using traffic data. Section III describes the prediction of the sector counts using the aggregate model. Section IV provides the selection of the best aggregate model from several alternatives using hypothesis testing. Section V describes the application of the method for predicting sector demand in Indianapolis Center. Conclusions and future work is described in Section VI.

II. Sector Flow Model

A sector flow model is developed in this section. The model can be used for predicting the sector demand, which is the number of aircraft in a given sector. The sector flow model follows the center flow model\textsuperscript{8} with some modifications. The sector flow model is expressed in the standard state space form\textsuperscript{12}. The state transition matrix was identified by the aircraft locations provided by ETMS. The state transition matrix is used to generate the sector demand prediction model.
For all the sectors in a given center, from one time step to the next, the inflow to the sector contains aircraft flying into the sector and departing from airports in the sector. The outflow from the sector contains the aircraft flying out of the sector and landing at airports in the sector. The sector flow behavior model can be described as in Fig. 2, where \( a_{i,j} \) for \( i \neq j \) is the number of aircraft in sector \( i \) flying to sector \( j \) at the next time step, \( a_{i,j} \) for \( i = j \) is the number of aircraft in sector \( i \) staying at the sector at the next time step, \( u_{i,\text{out}} \) is the number of aircraft in sector \( i \) flying out of the center or landing at the next time step, and \( u_{i,\text{in}} \) is the number of aircraft arriving from other centers to sector \( i \) at the next time step.

Assume there are \( N \) sectors in a given center, the sector flow of the center at time \( k \) is defined by the matrix

\[
T(k) = \begin{bmatrix}
    a_{1,1}(k) & \cdots & a_{1,N}(k) & u_{1,\text{out}}(k) \\
    \vdots & \ddots & \vdots & \vdots \\
    a_{N,1}(k) & \cdots & a_{N,N}(k) & u_{N,\text{out}}(k) \\
    u_{1,\text{in}}(k) & \cdots & u_{N,\text{in}}(k) & 0
\end{bmatrix}
\]  

(1)

The \( i^{th} \) row of \( T(k) \) represents the aircraft staying in, and all the outflow of sector \( i \) during the time interval from \( k \) to \( k+1 \). The traffic demand of sector \( i \) at time \( k \) can be obtained by the sum of the row, formulated as

\[
s_i(k) = \sum_{j=1}^{N} a_{i,j}(k) + u_{i,\text{out}}(k).
\]  

(2)

The \( i^{th} \) column of \( T(k) \) represents the aircraft staying in, and all the inflow to sector \( i \) from time \( k \) to \( k+1 \). The sector demand of sector \( i \) at time \( k+1 \) can be obtained by the sum of the column, formulated as

\[
s_i(k+1) = \sum_{j=1}^{N} a_{j,i}(k) + u_{i,\text{in}}(k).
\]  

(3)

From Eq. (2) and Eq. (3), the following equality holds

\[
\sum_{j=1}^{N} a_{i,j}(k+1) + u_{i,\text{out}}(k+1) = \sum_{j=1}^{N} a_{j,i}(k) + u_{i,\text{in}}(k).
\]  

(4)
The sector flow model can be written in the standard state space form. The state vector \( \mathbf{x}(k) \) is formed by the sector demands of all sectors in the center, \([s_1(k) \ \cdots \ s_N(k)]^T \). The state transition matrix \( \mathbf{A}(k) \) that maps \( \mathbf{x}(k) \) to \( \mathbf{x}(k+1) \) with an input term \( \mathbf{u}(k) = [u_{1,in}(k) \ \cdots \ u_{N,in}(k)]^T \) is defined as

\[
\mathbf{A}(k) = \begin{bmatrix}
    a_{1,1}(k)/s_1(k) & \cdots & a_{N,1}(k)/s_N(k) \\
    \vdots & \ddots & \vdots \\
    a_{1,N}(k)/s_1(k) & \cdots & a_{N,N}(k)/s_N(k)
\end{bmatrix}.
\] (5)

The off-diagonal \((i, j)\) element of matrix \( \mathbf{A}(k) \) represents the portion of sector demand at sector \( j \) flying to sector \( i \) at the next time interval, while the diagonal \((i, i)\) element of matrix \( \mathbf{A}(k) \) represents the portion of sector demand at sector \( i \) staying at the sector at the next time interval. Note that if \( s_j(k) = 0 \), \( a_{i,j}(k)/s_j(k) \) is replaced by 0. From Eq. (3) and Eq. (5), it can be shown that

\[
\mathbf{A}(k)\mathbf{x}(k) + \mathbf{u}(k) = \begin{bmatrix}
    \sum_{j=1}^{N} a_{j,1}(k) + u_{1,in}(k) \\
    \vdots \\
    \sum_{j=1}^{N} a_{j,N}(k) + u_{N,in}(k)
\end{bmatrix} = \begin{bmatrix}
    s_1(k+1) \\
    \vdots \\
    s_N(k+1)
\end{bmatrix} = \mathbf{x}(k+1). \] (6)

Therefore, the state space equation of the sector flow model is defined as the following

\[
\mathbf{x}(k+1) = \mathbf{A}(k)\mathbf{x}(k) + \mathbf{u}(k). \] (7)

The sector model developed in this section emphasizes the way data appears in ETMS and is similar to the center-level model. 

### III. Sector Demand Prediction Model

The sector demand prediction model in a given center uses the state transition matrix identified in the sector flow model, the observed sector demands, the estimated inflow and outflow from other centers, and the estimated arrival and departure in the center to perform the demand prediction. For a single step prediction, the model is described as

\[
\hat{\mathbf{x}}(k+1) = \mathbf{A}(k)\mathbf{x}(k) + \hat{\mathbf{u}}(k). \] (8)

where \( \hat{\mathbf{x}}(k+1) \) is the predicted sector demand vector at time \( k+1 \) with the observed sector demand vector \( \mathbf{x}(k) \), and \( \hat{\mathbf{u}}(k) \) is the estimated system input including the estimated aircraft departing and flying in from other centers at time \( k \).

For multiple steps prediction, the predicted sector demand vector is used as the observed sector demand vector in Eq. (8) at the next time step. Therefore, the \( p \)-steps ahead prediction model is described as

\[
\begin{align*}
\hat{\mathbf{x}}(k+1) &= \mathbf{A}(k)\mathbf{x}(k) + \hat{\mathbf{u}}(k), \\
\hat{\mathbf{x}}(k+2) &= \mathbf{A}(k+1)\mathbf{x}(k+1) + \hat{\mathbf{u}}(k+1), \\
&\vdots \\
\hat{\mathbf{x}}(k+p) &= \mathbf{A}(k+p-1)\mathbf{x}(k+p-1) + \hat{\mathbf{u}}(k+p-1).
\end{align*} \] (9)
The model input $\hat{u}(k)$ can be estimated either by the historical data or by monitoring the flight plans of the aircraft at surrounding centers and the scheduled aircraft departure times.

IV. Multiple Reference Models and Hypothesis Testing

The sector demand predictions in the previous section used a single reference model. It is possible to build several models by using appropriate historical data that capture the differences due to daily, weekly, monthly and seasonal variations. Assume there are $M$ reference models. A summary of the model selection process is depicted in Fig. 3. At a given time $k$, the outputs from model $1, 2, \ldots, M$ are compared with the actual sector demand to compute the error associated with each model. The best reference model combination at time $k$ is revised to form the new model combination based on hypothesis testing. In addition, this section describes an approach for combining multiple reference models that are weighted by the conditional probabilities for improving sector demand prediction.

![Diagram](image)

Figure 3. Reference model selection using hypothesis testing.

The recursive formula that computes the posterior probability of each hypothesis given the history of sector demands is summarized below. The recursive relation is modified from the formulation in Bertsekas and Ng. The following notation is used to define:

$x_k$ : Sector demand vector at time $t_k$,  
$X_k$ : Sector demand history up to time $t_k$,  
$H_l$ : The $l^{th}$ hypothesis,  
$f(x_k | X_{k-1}, H_l)$ : Conditional probability density function of the sector demand vector given sector demand history and hypothesis $H_l$,  
$F(H_l | X_k)$ : Posterior probability of hypothesis $H_l$ given sector demand history,  
$\pi_l$ : A priori probability that true hypothesis is $H_l$.

The posterior probability of each hypothesis can be obtained through Bayes' theorem, and it is generated recursively according to the following equations:
\[ F(H_t | X_k) = \frac{f(x_k | X_{k-1}, H_t) \cdot F(H_t | X_{k-1})}{\sum_{i} f(x_k | X_{k-1}, H_i) \cdot F(H_i | X_{k-1})} \]  

(10)

and for the initial condition,

\[ F(H_t | X_0) = \pi_l \]  

(11)

\[ \pi_l \] depends on the value placed on various models at the beginning of the testing process. If \( M \) models are assumed to be equally likely

\[ \pi_l = \frac{1}{M} \text{ for all } l \]  

(12)

If model \( L \) from the \( M \) reference models is preferred before testing begins, then the \( \pi_l \) values can be initialized as \( \pi_L = 0.99, \pi_l = (1 - 0.99) / (M - 1) \) for all \( l \neq L \). At time \( k \), the model which maximizes \( F(H_t | X_k) \) is selected as the best model.

The conditional probability density function \( f(x_k | X_{k-1}, H_t) \) of the sector demand vectors can be determined by considering the sector demand prediction model which includes the aircraft departure model \( \tilde{u}_t \) and can be modeled as,

\[ \tilde{u}_t(k) = \mu_t(k) + \omega_t(k) \]  

(13)

The departure model includes a deterministic component \( \mu_t(k) \) of the departures and a stochastic component \( \omega_t(k) \) of the departures. The deterministic component \( \mu_t(k) \) can be estimated from the historical mean departure rate in each sector within the center and count of aircraft flying in and out of the center. The stochastic component \( \omega_t(k) \) is assumed to be discrete time white noise sequence \( \omega_t(k) \sim N(0, W_t(k)) \). Note that the departures could also be modeled as Poisson random variables. However, it is computationally efficient to model the sector demands as Gaussian random process. The values of \( \omega_t(k) \) can be selected to reflect the conditions associated with different reference models using historical data. A large value of \( \omega_t(k) \) at time \( k \) may reflect the fact that model \( l \) at time \( k \) represents departures having large uncertainties.

For all hypothesis \( H_t \), it can be seen that the predicted sector demand vector \( \tilde{x}_t(k) \) at each time step has a Gaussian distribution, and the mean can be determined by the following equations,

\[ E[\tilde{x}_t(k+1) | x(k), H_t] = m_t(k+1), \]
\[ m_t(k+1) = A_t(k) \tilde{x}_t(k) + \mu_t(k), \]
\[ m_t(k+2) = A_t(k+1)m_t(k+1) + \mu_t(k+1), \]
\[ \vdots \]
\[ m_t(k+p) = A_t(k+p-1)m_t(k+p-1) + \mu_t(k+p-1). \]  

(14)

In general, the mean can be expressed by

\[ m_t(k+p) = \Phi_t[k+p,k]x_t(k) + \sum_{j=k}^{k+p-1} \Phi_t[k+p,j+1]u_t(j), \]

where the discrete transition matrix has the properties \( \Phi_t[k+1,k_0] = A_t(k)\Phi_t[k,k_0] \) and \( \Phi_t[k_0,k_0] = I \), and can be obtained directly as \( \Phi_t[k,k_0] = A_t(k-1)A_t(k-1)\cdots A_t(k_0) \).

The covariance of the sector demand prediction estimate at time step \( k+1 \) under hypothesis \( H_t \), which is \( P_t(k+1) = E[(\tilde{x}_t(k+1) - m_t(k))(\tilde{x}_t(k+1) - m_t(k))^T | x_t(k), H_t] \) can be propagated using the following equations,
Note that the mean $m_i(k + p)$ can be used as the predicted number of aircraft at time $k + p$ when $i^{th}$ model ($H_i$) is selected. The rule for choosing the correct hypothesis $H_i$ can be as simple as picking the hypothesis that maximizes $F(H_i | X_k)$. Difficulties arise when characteristic of real-time air traffic is similar to more than one hypothetical model and differences between the posterior probabilities are small.

An alternative approach to switching between reference models is adapted in this study. The predicted sector demand at time $k + p$ is estimated by combining the predictions from all aggregate models. The predicted number of aircraft is formulated as a linear combination of all predictions with each prediction scaled by its associated likeliness (posterior probability) to real-time air traffic. Let $\tilde{x}(k + p)$ be the aircraft count at time step $k + p$ predicted at time $k$, the prediction can be formulated by the following equation:

$$\tilde{x}(k + p) = \sum_{i=1}^{M} F(H_i | X_k) \cdot m_i(k + p).$$

In this study, a collection of aggregate models are developed such that each LDSM characterizes one day of the month. The following section presents the results of using the best aggregate model and a linear combination of LDSMs for sector demand prediction.

V. Results

This section presents results based on applying the aggregate flow models to predict sector demand for a number of sectors in Indianapolis Center (ZID), shown in Fig. 4. The major flows of ZID include the arrivals to Philadelphia International Airport (PHL), Ronald Reagan Washington National Airport (DCA), Chicago O’Hare International Airport (ORD), Detroit Metropolitan Wayne County Airport (DTW), and Cleveland-Hopkins International Airport (CLE), the departures from ORD and DTW, the westbound traffic of airway J80 from New York Center (ZNY) and Boston Center (ZBW), and the traffic to New York Terminal Radar Approach Control (N90). As shown in Fig. 4, ZID is divided into 10 super high, 4 medium high and 11 high altitude sectors. Figure 5 shows the sector demand and the 15-minute peak sector demand at sector ZID93 on August 3, 2007. Note that in the paper, a day is defined as 24 hours starting at four o’clock EDT since most of the aircraft departing on the previous day would have landed before the starting time. The blue dots in Fig. 5 represent the 15-minute peak sector demand during the day.

A total of thirty one aggregate flow models of ZID Center are constructed by using the ETMS data from August 1, 2007 to August 31, 2007, respectively. Each LDSM serves as a reference model which characterizes one day of air traffic flow in Indianapolis Center. The reference models are built from Eq. (8) and Eq. (9). The deterministic component $\mu_i(k)$ in Eq. (13) of each model input is set equal to the number of aircraft departing and landing and flying in and out of ZID on the corresponding reference day. The covariance matrix $\Sigma_i(k)$ of stochastic component $\omega_i(k)$ is set equal to a diagonal matrix. A diagonal element is set to one if the corresponding deterministic component is non zero. In addition, the uncertainty in the state transition matrix is modeled as discrete time white noise sequence with covariance matrix set equal to identity matrix. The stochastic component of the departure model is assumed to be independent of the uncertainty in the state transition matrix.
Next, consider the use of the aggregate flow models for demand prediction. There are 25 sectors in ZID and the aggregate flow models provide demand prediction for each of these sectors. The sampling rate of the model is 5 minutes. The mean and covariance of predicted sector demand vector conditioned on each hypothesis can be propagated by Eq. (14) and Eq. (15), respectively. Figure 6 shows the error between the actual sector demand at ZID76 on September 1, 2007 and the 15-minute ahead predictions generated by models 1, 11, 13, and 25. The results for other models are similar. The variations in posterior probability using hypothetical models 1, 11, 13, and 25 are shown in Fig. 7. The posterior probability of other hypotheses is similar to hypothesis 1 and close to zero for the entire interval. The posterior probability of each hypothesis is computed recursively using Eq. (10), Eq. (11), and Eq. (12). Figure 6 and 7 show that air traffic flow before 9 a.m. on September 1, 2007 is similar to those characterized by models 11, 13 and 25. The posterior probability of hypothesis 11 converges to a high value close to 1 after 9 a.m. that day.

Figure 4. Superhigh (red), medium high (green) and high sectors (blue) in ZID center.

Figure 5. Sector demand and peak sector demand at sector ZID93 on August 3, 2007

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The absolute error between the actual sector demand and the conditional mean provided by an aggregate model is compared every 5 minutes. The mean of this error for the entire day is calculated for the 25 sectors in ZID. The average of mean absolute error for the Indianapolis Center is calculated for each aggregate flow model. The average prediction error on September 1, 2007 generated by each of the 31 models is shown in Table 1. In addition, the error of prediction that is generated by a linear combination of the mean expected value of the sector count for each of the 31 models multiplied by the posterior probability of the model using Eq. (16) is also shown. The range of mean absolute errors for the 31 aggregate sector flow models is between 1.21 and 1.63 aircraft. Obviously,
identifying the correct aggregate model can significantly improve the accuracy of the predictions. Model 11 has the smallest mean absolute error among the 31 models. The hypothesis testing algorithm correctly identifies model 11 as the best model for that day since it maximizes the posterior probability. Furthermore, the prediction that used a linear combination of probability-weighted sector count from each of the 31 models has a smaller error than model 11. Note that the real air traffic flow is similar to those characterized by models 13 and 25 in the morning. The combined model enhances the accuracy of the predictions by adapting model 13 or model 25 in early morning. Figure 8 shows the posterior probability of hypothesis 11 and 25 between 7 and 10 o'clock in the morning. The posterior probability of any one hypothesis is not equal to one which indicates that the predictions are combined from hypothetical models 11 and 25 between 8 and 9 o'clock that day. The results show that using a combination of probability-weighted predictions from several aggregate models improves the prediction accuracy.

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<td>1.41</td>
<td>1.20</td>
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</table>

The averages of mean absolute prediction error in Indianapolis Center on September 1, 2007 using a collection of models constructed from August 1, 2007 to August 31, 2007 data. The unit is the number of aircraft.

![Figure 8. Posterior probability of hypothesis 11 and 25 between 7AM and 10AM EDT.](image)

The models built from August, 2007 data are used to perform sector demand prediction for the month of September, 2007. The predicted demand at each sector is calculated using Eq. (16). The actual combination of models used varies with the day and time the demand is predicted. Since the traffic flow management decisions are made by comparing the peak number of aircraft in a fifteen-minute interval with sector's Monitor Alert Parameter (MAP) value, the 15-minute peak sector demand, defined as the maximum number of aircraft every 15 minutes, is used to measure the accuracy of the sector demand predictions. The results of 15-minute prediction at ZID93 on September 1, 2007 are shown in Fig. 9a. The Root-Mean-Square (RMS) error between the actual demand and the 15-minute predicted demand is equal to 1.92. For the 2-hours prediction, the result is shown in Fig. 9b. The RMS error is equal to 2.00. The average RMS error results of 15-minutes to 2 hours predictions at ZID76 and ZID80 for the month of September, 2007 are shown in Fig. 10. ZID80 has smaller average RMS errors than ZID76. The sector demand prediction errors vary only slightly with the look ahead time. The prediction results for the super high, medium high and high altitude sectors in ZID are summarized in Table 2. Over the month of September 2007, the RMS errors between the actual 15-minute peak sector demand and the predicted peak demand ranged between 1.79 and 2.64 on average.
Figure 9. Peak sector demand and predicted peak demand on September 1, 2007

Figure 10. Variation of prediction errors in two sectors in Indianapolis Center.

Table 2. Average prediction RMS error in Indianapolis Center over September, 2007 using a collection of probability-weighted models constructed by August, 2007 data. The unit is the number of aircraft.

<table>
<thead>
<tr>
<th>Name</th>
<th>MAP</th>
<th>Average prediction RMS error</th>
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<th>MAP</th>
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<td>ZID75</td>
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VI. Concluding Remarks

An air traffic sector demand forecasting model using an aggregate sector flow model was presented. The model is capable of both short-term (15 minutes) and mid-term (30 to 120 minutes) sector demand prediction with an average root-mean-squared error between 1.79 and 2.64 aircraft. Unlike trajectory-based models, the accuracy of these models do not vary with the prediction interval and the models are less susceptible to disturbances in the system. The state transition matrix in the model captures the air traffic flow property; therefore there is no need to store the individual aircraft trajectories in the system. Furthermore, a collection of aggregation models is constructed such that each may represent the traffic flow under different conditions. It is demonstrated that the best model for the actual traffic flow characteristic, is correctly identified based on hypothesis testing.

The usefulness of the demand prediction models depends on how they are used by the operations or the planning staff to make TFM decisions. The enhanced predictions from ETMS and the aggregate models proposed in this paper provide two different demand forecasts to the decision-makers. The ETMS demand predictions can be characterized as predictions based on the known TFM plans at the beginning of the prediction interval whereas the aggregate sector flow models provide demand predictions with built-in historical TFM responses. The demand predicted by the aggregate sector flow model dynamics can be modified by retaining the state transition matrix based upon historical data and modifying the aircraft departure data based on the known TFM plans as is done in current ETMS demand predictions.

References