Airline and Service Provider Collaborative Algorithms for Flight Route and Delay Decisions

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In this paper, algorithms are presented that model increased collaboration between the air traffic service provider and airspace users on flight route and delay decisions. These decisions are part of the traffic flow management function that constrains demand below capacity. Currently, users cannot make changes to the route or delay of a flight close to or after departure time and instead must send requests to the service provider who attempts to accommodate the users based on congestion and workload limitations. To mitigate this limitation, the algorithms model a new collaboration scheme. First, users directly implement their flight route and delay decisions, when the flight is further from the congested airspace, without sending a request to the service provider. The service provider can override their action when the flight becomes closer to the congested airspace. Second, users send flight ranking, route ranking and location-to-absorb-delay preferences to the service provider. The service provider may reject these preferences if needed. The algorithms are used to study whether increasing users’ responsibility and increasing their preferences would prevent maintaining demand below capacity. To prevent demand from exceeding capacity the algorithms impose limits, such as available routes and imposed flow rates, on user decisions. A simulation case demonstrates the impact of the collaboration schemes on reducing demand below capacity within an en-route center. Preliminary results indicate that aircraft delay and, to a larger extent, passenger delay are reduced. However, congestion is reduced by a smaller amount when user preferences are considered by the service provider. Giving users responsibility according to service provider limits and delay feedback did not increase congestion.

Nomenclature

\begin{align*}
\alpha_1, \alpha_2, \alpha_3, \alpha_4 & = \text{Airline route cost function coefficients} \\
Buffer_k & = \text{Arrival time buffer for alternative route } k \\
c_{j,k,t} & = \text{Capacity of sector } j \text{ during the time period } t \text{ that route } k \text{ is projected to demand sector } j \\
CF_k & = \text{Cost function for route } k \\
CR & = \text{Connecting rate at the destination airport} \\
D_a & = \text{Total ground and airborne delay required for sectors along the assigned route} \\
D_k & = \text{Total ground and airborne delay required for sectors along alternative route } k \\
ETA_a & = \text{Unimpeded estimated time of arrival to the destination for the assigned route}
\end{align*}

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\[ \begin{align*}
ET_{A_{k}} &= \text{Unimpeded estimated time of arrival to the destination for alternative route } k \\
Fuel_{k} &= \text{Fuel burn for route } k \\
I_{\text{AOCimpact},k} &= \text{An airline route switch factor index representing additional cost to connecting flights for route } k \\
I_{\text{fuel},k} &= \text{An airline route switch factor for fuel burn cost index for route } k \\
I_{\text{NASimpact},k} &= \text{An airline route switch factor that measures the impact of this flight on congestion in the airspace when using route } k \\
I_{\text{sat},k} &= \text{An airline route switch factor that measures customer satisfaction for route } k \\
N_{l} &= \text{Number of congested sectors along the route (air traffic service provider criteria)} \\
N_{j,k} &= \text{Binary value that equals 1 if sector } j \text{ is congested for route } k \\
N_{k} &= \text{Number of congested sectors along route } k \text{ (airline operation center criteria)} \\
N_{\text{peak}} &= \text{Maximum value of the demand minus capacity across all time periods} \\
P_{\text{ATS}} &= \text{Time parameter set by the air traffic service provider to minimize switching to an alternate route if the time savings is not sufficiently large} \\
P_{c} &= \text{Threshold for sectors close to capacity for airline operation centers considering airspace congestion impacts during route ranking} \\
q_{j,k,t} &= \text{Demand of route } k \text{ for sector } j \text{ during time period } t \\
R_{j} &= \text{Sector at index } j \\
S_{k} &= \text{Scaled delay for route } k \\
U_{k} &= \text{Utility for route } k \\
\end{align*} \]

I. Introduction

In the current National Airspace System, the traffic flow management function of balancing demand and capacity is centralized and provided by the Federal Aviation Administration, whose Air Traffic Control System Command Center develops strategic traffic flow management initiatives. The users of the National Airspace System, which include airlines, general aviation, and business aviation, are impacted by these plans, but their involvement in the traffic flow management decision-making process is still limited. Collaborative decision-making is an initiative currently in operation that originated from the Federal Aviation Administration’s airline data exchange program of 1993. Collaborative decision-making improves traffic flow management plans by increasing data exchange and user involvement, particularly by swapping flight priorities during ground delay programs and airspace flow programs imposed by the Command Center. However, user participation is particularly lacking in local situations which do not require involving the Command Center and where users would have to coordinate directly with a local traffic management facility. As a result, users do not always receive the desired, timely and certain options from the air traffic service provider and user preferences and requests are not adequately considered due to high service provider workload. The outcome is that the traffic flow management solutions provided by the service provider are more restrictive and the airlines are passive rather than proactive in providing information and requesting preferences. Therefore, traffic flow management can be improved by increasing the collaboration between the service provider and the airspace users, particularly in local situations.

Previous work proposes increasing the involvement of users in the traffic flow management decision-making process above the level of collaborative decision-making and to additionally include local situations to realize further benefit. One concept allows users to send a prioritized list of alternative routing options, which the traffic managers incorporate in reroutes assigned to flights. An agent-based model was used to conclude that the service provider cannot make the best traffic flow management decision without collaboration from airlines. However, if users make decisions independently, they cause excess congestion. Idris et al. proposed a collaborative traffic flow management concept that dynamically allocates some of the responsibility, both in selecting and in implementing traffic flow management plans, from the service provider to airline operation centers.

This paper describes algorithms that model increased user involvement in traffic flow management, including (1) the users making flight route and delay decisions at a distance from the congested airspace sectors, while the service provider performs this function closer to these sectors overriding the user actions if needed, and (2) incorporating user preferences into the service provider flight route and delay decisions when the service provider is responsible and if the preferences are acceptable. User preferences considered include: priorities between flights, priorities between routes, and desired location to absorb delay. With the increased user responsibility and scope of preferences, the users may be better able to meet their objectives; however the problem of demand exceeding capacity may not be solved. Therefore, the user responsibility is allowed within certain limits: independent user flight changes may only occur beyond a distance threshold from the congested airspace and must conform to a plan.
specifying available routes and corresponding flow rates. In addition the service provider shares the delay expected to maintain demand below capacity which the users base their decisions on. Also the service provider can reject user preferences if needed. The completeness of information and user intent is modeled to reflect that the users make decisions independently and without coordination. Due to paper length limitations, only one set of parameters is analyzed in terms of maintaining demand below capacity at a localized en-route sector. Preliminary results showed that allowing users responsibility to directly implement flight route and delay decisions, according to the service provider limits and delay feedback, did not increase demand further above capacity, while users benefited through reduced delay.

First, Section II presents a comparison of the service provider and user algorithms for the overall process of flight route and delay decisions, priority ranking among flights, switching between routes, and selecting the location to absorb delay. A congested en route airspace is analyzed with the proposed collaboration scheme in Section III. The paper concludes in Section IV with a summary of simulation results and future research to refine the models.

**II. Collaboration Algorithms for Flight Route and Delay Decisions**

Flight route and delay decision responsibility is allocated to the air traffic service provider (ATSP) or airline operations center (AOC) depending on the distance from the sectors with demand projected to exceed capacity, shown as a red ellipse in Fig. 1.

Two airspace regions are modeled as shown in Fig. 1 (on two sides of the blue-green vertical lines) with two different collaborative actions. For both airspace region 1 on the left and airspace region 2 on the right a set of inputs (Block 0) is sent from the ATSP to the AOCs. The inputs in Block 0 include a list of congested sectors, the flights that demand these congested sectors, a delay map which is explained later in this section, the available alternative routes, and the rates along these routes.

Further from sectors with demand projected to exceed capacity, shown in airspace region 1 on the left in Fig. 1, collaboration is achieved by assigning AOCs decision-making responsibility to switch routes and absorb delays for their flights (Block 1). There is no ATSP action in this airspace region after the AOCs make flight route and delay decisions.

Collaborative actions in airspace region 2 close to the sectors with demand projected to exceed capacity are shown on the right in Fig. 1. They include: the AOCs sending flight priority ranking, route ranking, and a desired location to absorb delay preferences to the ATSP (Block 2) and the ATSP accounting for these preferences when

![Figure 1. Overview of collaboration algorithms.](image-url)
ranking flights (Block 3) and incrementally switching routes and assigning delay for each flight (Block 4). For AOCs that don’t take responsibility, the collaboration scheme does not change based on distance from the sectors with demand projected to exceed capacity and is according to the collaboration scheme close to the constraint (Blocks 2, 3, and 4). The location representing the change in ATSP and AOC responsibility is currently distance-based and set at sector boundaries. An alternative would be time-based transitions based on the projected time of entry to the congested sectors. This dynamic allocation of traffic flow management (TFM) responsibility is based on a previously developed collaborative traffic flow management (CTFM) concept of operations.\textsuperscript{9,10,13}

The inputs to the algorithm are first described in Subsection A which corresponds to Block 0 in Fig. 1. Subsection B describes the overall process for the ATSP (Blocks 2, 3, and 4) and AOCs (Block 1). The ranking of flights described in Subsection C relates to Block 3 for the ATSP and Blocks 1 and 2 for the AOCs. Subsection D describes the ranking of routes that are used in Block 4 for the ATSP and Blocks 1 and 2 for the AOCs. The preferred location to absorb delay is then described in Subsection E which is used by the ATSP in Block 4 and by the AOCs in Block 2 when generating preferences. The focus of the current algorithm is sectors which can be extended to other airspace resources such as fixes and airports while keeping the overall structure.

A. Inputs from other Traffic Flow Management Activities

It is assumed that other TFM activities provide inputs for the flight route and delay decisions as shown in the dashed box in Fig. 1 (Block 0). One of these inputs is the identification of sectors with demand projected to exceed capacity and includes a list of sectors with demand projected to exceed capacity, the start and end times capacity is exceeded, and a list of flights demanding the sectors during the times capacity is projected to be exceeded. Another input is a list of flights that each AOC has decision-making responsibility for based on distance from the constraint.

ATSP delay feedback to the AOCs is an input that specifies the average delay predicted to be absorbed in each sector during each 15 minute period over a time horizon. The delay map is generated by assigning delays to reduce demand below capacity using the currently assigned routes for each flight with no rerouting. The delay map does not currently distinguish which congested sector caused the delay that is being absorbed by a flight in a sector. Delay is absorbed upstream of a congested sector but the downstream congested sector being avoided may be different for different flights. However, this form of a delay map provides a reasonable approximation, if several sectors are common.

Lastly, the ATSP and AOCs switch routes for a flight and assign delay consistent with a flow plan, which is an input that specifies the alternative routes and the rates that are needed to reduce demand below capacity.

B. Overall Process

This section describes differences between the ATSP and AOC overall process for route and delay decisions which corresponds to Blocks 2, 3, and 4 for the ATSP and Block 1 for the AOCs in Fig. 1.

1. ATSP Responsible for Flight Route and Delay Decisions

The ATSP has responsibility for flight route and delay decisions for flights close to the sectors with demand projected to exceed capacity and all other flights where the AOC does not take responsibility. When the ATSP has responsibility for flight decisions the ATSP can explicitly reduce demand below capacity for sectors with a demand projected to exceed capacity since the process is incremental (one flight at a time) and at each increment the ATSP can assign delay so that sector capacity is not exceeded.

The sequence of activities for the ATSP begins by sending to each AOC a list of flights that is filtered so that the AOCs only receive information regarding their own flights. The AOCs respond with preferences and flights that the AOC is taking responsibility for and has made route and delay decisions for. When the ATSP receives the list of flights from the AOCs, the ATSP decides whether or not to override the AOC decisions based on the current position of the flight since the flight may now be close to the sectors with demand projected to exceed capacity and the ATSP has responsibility. Then the ATSP generates a priority ranked list of flights, considering AOC preferences, to be used in the incremental decisions for switching routes and assigning delays to flights. For each flight in the ranked list the ATSP considers AOC preferences for switching to an alternate route and preferred location to absorb delay. When calculating delay for a flight to maintain demand below capacity the ATSP only considers demand from higher ranked flights.

The ATSP decisions are made periodically according to a configurable parameter with a default value of 15 minutes. Each time the ATSP algorithm is run the simulation is updated using the calculated delays and assigned routes. Namely, a new list of sectors with demand projected to exceed capacity along with a new list of flights demanding these sectors. The delay map and flow plan are also updated.
2. **AOCs Responsible for Flight Route and Delay Decisions**

When AOCs make flight route and delay decisions the balancing of demand and capacity for each sector is challenging due to the presence of several AOCs making decisions simultaneously and independently without coordination. Whereas there is only one ATSP agent, or alternatively several ATSP agents coordinating to effectively act as one agent with a single objective, there may be several AOC agents, each with different objectives. The AOC objective is the combined effect of all preferences in terms of: the priority ranking of flights, the priority ranking of routes, and the preferred location to absorb delay relative to the constraint. The AOCs also have risk tolerances different from the ATSP and may be more aggressive than the ATSP by not moving aircraft away from congested regions of airspace. Each AOC does not know the intention and action of other AOCs in terms of how many flights will be moved away from the sectors with demand exceeding capacity and to where, since the AOC decisions are made simultaneously and without coordination in the current model.

The AOC decision algorithm presented is simplified and based on the concept that the AOCs will switch to less congested routes in response to the ATSP providing delay estimates for sectors that demand projected to exceed capacity. The AOCs do not explicitly reduce sector demand below capacity since without coordination the availability of a sector is generally not known. More specifically, AOCs select a route and decide on the delay to be absorbed along the route using the ATSP delay feedback in their utility function. Since the AOCs utility functions are based on ATSP delay feedback the selected route will tend to avoid congestion because the ATSP delay estimate is based on balancing demand and capacity. The delay absorbed on the other hand is strictly according to the ATSP delay feedback. This is a conservative initial model for independent decisions by AOCs, which could be modeled differently so that AOCs decide on the delay according to their own utility. Also, the ATSP delay feedback is updated periodically to partially account for other AOC actions. A National Airspace System (NAS) impacts parameter in the AOC route selection algorithm can be modified to model the degree of aggressiveness for an AOC to avoid congested sectors.

C. **Priority Ranking of Flights**

A priority ranked list of flights is required as an input to both the ATSP and AOC decision algorithms. The ATSP uses the priority ranked list of flights to decide the order that flights are considered for switching to alternative routes and assignment of delay. This order is determined in Block 3 in Fig. 1. For the highest ranked flight the demand from other flights is not considered so no delay is assigned to this flight. Conversely the demand from higher ranked flights must be considered when making route switch decisions and assigning delay for lower ranked flights since the higher ranked flights make first use of the available capacity. A description of the ATSP priority ranking of flights and incorporation of AOC flight ranking preferences is presented first, followed by an AOC algorithm that ranks flights differently. The AOCs directly implement their flight ranking when they have responsibility (Block 2 in Fig. 1) send the flight ranking as a preference to the ATSP when the ATSP has responsibility (Block 2 in Fig. 1).

1. **ATSP Priority Ranked List of Flights**

The generation of a priority ranked list of flights by the ATSP begins with a pool of flights and first ranks airborne flights higher than flights on the ground, which allows for more ground delay in the cases where airborne demand approaches capacity. Then the assigned route for each flight is used to determine the priority among flights on the ground and among flights in the air, separately, based on First-Come First-Served (FCFS).

The FCFS ranking is measured by estimated time of arrival (ETA) to the first sector with a demand projected to exceed capacity along the aircraft’s route. To be considered as a sector with capacity exceeded, the time that the aircraft is projected to occupy the sector must occur when capacity is projected to be exceeded. The ETA for an aircraft is measured at the entry point to the sector. Using entry times at the first sector with a demand projected to exceed capacity may not be the most equitable method to rank flights and is a subject for further research.

The ATSP FCFS flight ranking is modified to incorporate AOC flight ranking preferences as follows:

a. Start with the ATSP FCFS ranked list of flights.

b. Pick randomly one AOC and obtain the priority among its flights. The random selection prevents systematic bias in the absence of a more refined flight ranking method.

c. For that AOC, swap higher priority flights that were located below in the pool with lower priority flights that were located above in the pool. This is done by locating the flight with highest priority in the AOC priority list, and looking up the next flight of that airline above it in the pool with lower AOC priority. Compare the projected times of entry to the first sector with demand projected to exceed capacity and if these differ by more than a threshold time then do not grant the swap. This threshold is a configurable parameter with a default value of 30 minutes to avoid excessive position shifting. Otherwise, the algorithm swaps both aircraft if one of the following two conditions is met:
Condition 1: There is no aircraft from another AOC ranked between them in the pool. The change can be granted because it does not affect any other AOC. Condition 2: There are aircraft from other AOCs between them in the pool. Table 1 enumerates the rules for allowing changes. Consider flights “a” and “b” from the same AOC and “x” is a flight from a different AOC, and the ATSP flight ranking order in the pool is initially b>x>a, but preferences are such that “a” has higher priority than “b”. Then, the change in the order of the list is granted according to Table 1. The rules are intended to deny a swap between flights if there is an impact on a second airline.

d. If the change is not granted, the algorithm continues sorting flights in the ATSP FCFS ranking pool until all flights achieve their preferred position according to the AOC flight ranking or the changes are rejected.

2. AOC Priority Ranked List of Flights

The AOC priority ranked list of flights is sent as a preference to the ATSP when the ATSP has responsibility for flight route and delay decisions. The AOC directly implements the preferred flight ranking when the AOC has responsibility. One difference between the ATSP and AOC flight ranking algorithms is that the AOC ranks its own flights only whereas the ATSP ranks all flights using a FCFS criterion.

Another difference is the ranking method. The AOC ranking algorithm is currently only based on the number of passengers aboard the aircraft: the higher the number of passengers, the higher the priority in the list of flights. The number of passengers is estimated using the number of seats and historical load factors. The load factors are based on the Bureau of Transportation Statistics DB1B database‡‡. This criterion is independent of the incurred delay of each aircraft so the AOC priority between flights is unchanged during the scenario. The AOCs consider factors other than the number of passengers on an aircraft so the AOC flight ranking algorithm could be enhanced in future research.

D. Ranking of Routes and Route Switching Logic

The behavior of the ATSP for switching a flight to an alternative route is first presented (Block 4 in Fig. 1) followed by the AOCs decision-making in the priority ranking of routes and switching to alternative routes. The AOC route decision-making is directly implemented when the AOCs have responsibility (Block 1 in Fig. 1) and sent to the ATSP as a preference when the ATSP has responsibility (Block 2 in Fig. 1). The ATSP logic for switching routes considers AOC route ranking preferences. Alternative routes are an input from the flow planning activity.

1. ATSP Ranking of Routes and Route Switching Logic

The ATSP algorithm for route switching considers three factors: the congestion of sectors along the route, route travel time, and the AOC preference if one was sent. Congestion is given a higher priority than travel time or AOC preference so that the ATSP will always switch a flight from a more congested to a less congested route. Only if two routes are uncongested or equally congested will the ATSP consider travel time or the AOC preference. The steps for calculating congestion along a route are as follows:

Step 1: Initialize $N_c$, the number of congested sectors along the route, to zero. Also, generate a list of sectors along the route.

Step 2: Select and remove the next $R_s$, the sector at index $j$ along the route, from the list of sectors.

Step 3: For each time increment of $R_s$ that the flight is projected to occupy, calculate the value of demand minus capacity of the sector. Since the ATSP is doing incremental assignment of all flights the demand is only from higher ranked flights. Define $N_{peak}$ as the maximum value of the demand minus capacity across all time periods.

Step 4: If $N_{peak}$ exceeds the capacity of the sector by greater than a threshold for congestion then increment $N_c$. This threshold is currently set to one, or demand at a level of one aircraft below capacity, based on the ATSP maintaining demand below capacity in simulation experiments.

Table 1. Rules to allow flight ranking changes between flights operated by different AOCs.

<table>
<thead>
<tr>
<th>b shares a sector with x</th>
<th>x shares a sector with a</th>
<th>b shares a sector with a</th>
<th>Change granted?</th>
</tr>
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<tbody>
<tr>
<td>yes</td>
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‡‡ http://www.transtats.bts.gov/Tables.asp?DB_ID=125
Step 5: If all sectors along the route have not been examined return to Step 2, otherwise continue to Step 6. Step 6: Return $N_c$, the number of congested sectors along the route, as a congestion measure for the route.

If the $N_c$ value for an alternative route is less than the $N_c$ value for the currently assigned route then the ATSP will switch the flight to the alternative route. Otherwise, if the $N_c$ values are equal then a route switch rule based on delay or AOC preferences is considered. If an AOC route preference for the flight is sent from the AOC to the ATSP then the AOC preference is used, otherwise the ATSP will rely on delay to switch to an alternative route. More explicitly, if there are two candidate routes that are not congested or equally congested and one route is ranked higher than the other route by the AOCs then the ATSP will switch to the higher ranked route based on the AOC preference criteria.

For the delay criteria, the ATSP switches a flight to an alternative route if the travel time difference between the currently assigned route and an alternative route exceeds an ATSP delay threshold parameter as shown in Eq. (1).

$$\left(\text{ETA}_a + D_a\right) - \left(\text{ETA}_k + D_k\right) > P_{\text{ATSP}}$$

where: $\text{ETA}_a$ is unimpeded (without ground or airborne delay) arrival time to the destination for the assigned route, $D_a$ is the total ground and airborne delay required to keep demand at or below capacity for sectors along the assigned route, and $P_{\text{ATSP}}$ is a time parameter set by the ATSP to minimize switching to an alternate route if the time savings is not sufficiently large. $\text{ETA}_k$ and $D_k$ are defined similar to $\text{ETA}_a$ and $D_a$ for alternative route $k$.

Figure 2 shows an example of the ATSP route switching logic with three routes and $P_{\text{ATSP}}$ set to 10 minutes. The number of congested sectors along the route is first calculated followed by the travel time difference between the alternative routes (route 2 and 3) and the assigned route (route 1). Since route 1 has more congested sectors (2 sectors) compared to routes 2 and 3 (1 sector) the ATSP switches this flight from route 1 to route 2 and route 2

![Figure 2. ATSP switching to an alternative route example.](image-url)
becomes the assigned route. Route 1 is no longer a candidate due to the congestion criteria. Route 3 has the same number of congested sectors as the assigned route 2 so the delay criteria is now tested (assuming no AOC preference). Route 3 arrives to the destination 15 minutes earlier than route 2 based on 20 minutes of delay required along route 2 to maintain demand below capacity. This 15 minutes difference is larger than the threshold of 10 minutes so the flight is switched from route 2 to route 3 and route 3 becomes the assigned route. There are no other alternative routes that meet the criteria for congestion or delay so the route switch evaluation for this flight ends.

2. AOC Ranking of Routes and Route Switching Logic

The AOC route ranking algorithm is used to generate preferences as an input to the ATSP flight route and delay decision-making when the ATSP has responsibility. The AOC always assigns a flight to the highest ranked route when the AOC has responsibility.

Utility theory is used as a basis for the AOC route ranking algorithm. The focus is on the ability to model different AOC behaviors in terms of: the factors that AOCs consider, aggressiveness, and risk tolerance. The utility model is specified by examining 20 factors that AOCs consider in their flight decision-making. This list of factors is reduced based on data availability and likelihood of occurrence. The remaining factors are grouped into four indices as shown in Eq. (2) and used to specify a cost function with a description of each of the indices.

$$ CF_k = \alpha_1 I_{\text{fuel},k} + \alpha_2 I_{\text{AOCimpact},k} + \alpha_3 I_{\text{sat},k} + \alpha_4 I_{\text{NASimpact},k} $$

(2)

where \( k \) represents a route, \( CF_k \) is the cost function, \( I_{\text{fuel},k} \) is a fuel burn cost index, \( I_{\text{AOCimpact},k} \) is an index representing additional cost to connecting flights, \( I_{\text{sat},k} \) is a customer satisfaction index, and \( I_{\text{NASimpact},k} \) is the impact of this flight on congestion in the NAS. The coefficients \( \alpha_1, \alpha_2, \alpha_3, \) and \( \alpha_4 \) that define AOC behavior are specific to an AOC. However, the modeling approach has been to group the AOCs by type such as mainline legacy, low cost, and regional and specify coefficients for the group rather than specifying different coefficients for each AOC agent. To be consistent with utility theory the cost function is scaled so that the smallest value is the worst and the largest value the best according to Eq. (3).

$$ U_k = \left( \frac{CF_k - CF_{\text{WORST}}}{CF_{\text{BEST}} - CF_{\text{WORST}}} \right) $$

(3)

where \( U_k \) is the utility for route \( k \), \( CF_{\text{BEST}} \) is the best (lowest) cost which should generally be zero, and \( CF_{\text{WORST}} \) is the worst (highest) cost for an alternative route.

The fuel burn cost index, \( I_{\text{fuel},k} \), is obtained by normalizing to a range of [0,1] the estimated fuel burn from the current position of the flight to the destination as shown in Eq. (4). Higher values of the fuel burn cost index are considered worse.

$$ I_{\text{fuel},k} = \frac{\text{Fuel}_k - \text{Fuel}_{\text{min}}}{\text{Fuel}_{\text{max}} - \text{Fuel}_{\text{min}}} $$

(4)

where \( \text{Fuel}_k \) is the fuel burn for route \( k \), \( \text{Fuel}_{\text{min}} \) and \( \text{Fuel}_{\text{max}} \) are respectively the minimum and maximum fuel burns across all alternative routes. In the cases where minimum and maximum fuel burns are equal then \( I_{\text{fuel},k} = 0/0 = 1 \) is chosen as a convention.

The index \( I_{\text{AOCimpact},k} \) considers the level of disruption that delaying a flight can have on other flights. Maintaining a level of delay is important for flights scheduled to large hub airports with a high percentage of connecting passengers. For this calculation a buffer is first defined as the difference between the ETA to the destination airport and the scheduled time of arrival to the destination airport. This buffer is then used to specify the AOC impact index in Eq. (6).

$$ Buffer_k = ETA_k - ETA_0 $$

(5)

$$ I_{\text{AOCimpact},k} = \frac{Buffer_k - Buffer_{\text{min}}}{Buffer_{\text{max}} - Buffer_{\text{min}}} $$

(6)

where \( Buffer_k \) is the arrival time buffer for alternative route \( k \), \( ETA_k \) is the ETA for route \( k \), and \( ETA_0 \) is the scheduled time of arrival to the destination airport. \( Buffer_{\text{min}} \) and \( Buffer_{\text{max}} \) are respectively the minimum and maximum values of \( Buffer_k \) across all alternative routes. In the cases where the minimum and maximum buffer are equal \( I_{\text{AOCimpact},k} = 0/0 = 1 \) is chosen as a convention.

The long-term benefit of on-time performance is used to specify the customer satisfaction index, \( I_{\text{sat},k} \), for a route \( k \). On-time performance is positively correlated with customer satisfaction, airline profit, and airfare. Long flight delays decrease customer satisfaction, especially for connecting passengers. To simplify the index the FAA’s 45
minute rule, where the FAA requests that at least 45 minutes are scheduled for passenger connections, is used as a criterion. The assumption is that flight delay of more than 45 minutes affects both connecting passengers and non-stop passengers, flight delay between 15 and 45 minutes mostly affects connecting passengers. Flight delay less than 15 minutes are not included since the FAA considers less than 15 minutes of delay as on-time. The customer satisfaction index, $I_{sat}$, is defined in Eq. (8) based on the definition of scaled delay in Eq. (7).

$$S_k = \begin{cases} 
D_k & \text{if } D_k \geq 45 \text{min} \\
D_k C_R & \text{if } 15 \text{min} \leq D_k < 45 \text{min} \\
0 & \text{otherwise}
\end{cases}$$

(7)

$$I_{sat} = \begin{cases} 
S_k / S_{\text{max}} & \text{if } S_{\text{max}} > 0 \\
0 & \text{if } S_{\text{max}} = 0
\end{cases}$$

(8)

where $D_k$ is the delay along route $k$, $C_R$ is the connecting rate at the destination airport, $S_k$ is the scaled delay for route $k$, and $S_{\text{max}}$ is the maximum value of $S_k$ across all alternative routes. The connecting rate, $C_R$, was obtained from the Bureau of Transportation Statistics based on the ratio of the arriving passengers at an airport that are connecting to the total arrivals at an airport.

The NAS impact index, $I_{\text{NASimpact}}$, is an aggressiveness indicator of whether the AOCs take into account the level of congestion of sectors along the route. The NAS impact index is used to control how much the AOC considers ATSP concerns to lower the risk of a proposed route being rejected. The index is based on the total amount demand exceed capacity along the route as shown in Eq. (9). A count of these sectors exceeding the threshold is then made in Eq. (10) with a peak of this count used to scale the NAS impact index in Eq. (11).

$$N_{j,k} = \begin{cases} 
\sum_i \left[ q_{j,k,t} - \left( c_{j,k,t} - P_c \right) \right] & \text{if } q_{j,k,t} > c_{j,k,t} - P_c \\
0 & \text{otherwise}
\end{cases}$$

(9)

$$N_k = \sum_j N_{j,k}$$

(10)

$$I_{\text{NASimpact},k} = \begin{cases} 
N_k / \max_i \{ N_k \} & \text{if } \max_i \{ N_k \} > 0 \\
0 & \text{otherwise}
\end{cases}$$

(11)

where the congestion $N_{j,k}$ is calculated using $q_{j,k,t}$ and $c_{j,k,t}$, respectively the demand and capacity of sector $j$ during the time periods $t$ that route $k$ is projected to demand sector $j$. $N_{j,k}$ is also based on the threshold for sectors close to capacity $P_c$. The total amount demand exceeds capacity along route $k$ is referred to as $N_k$ which is used to define the NAS impact index $I_{\text{NASimpact},k}$ based on the ratio to the peak congestion for route $k$.

The AOC consideration of NAS impacts differs from the ATSP in two key aspects. First, for the ATSP the congestion of a route is considered a hard constraint and will always switch a flight from a more congested to a less congested route. Conversely, the AOCs consider NAS impacts as a factor in their utility function that influences their decision but can ultimately be overridden if the other factors are given a higher weight. Second, the ATSP has more accurate information regarding the congestion of sectors since the ATSP makes incremental flight decisions and knows immediately the impact on congestion of switching the route for a flight. The AOCs must rely on congestion information from a previous iteration which may be out of date. The congestion information used by the AOCs is current during the generation of the route ranking preference but out of date when the ATSP is making route decisions because the route ranking preference is generated before the incremental flight route and delay decisions which have an impact on congestion. The AOCs switch routes and assign delays to their flights in parallel so updated congestion information based on changes from one AOC will not be known by other AOCs until the ATSP broadcasts congestion information again.

E. Location to Absorb Delay Relative to a Constraint

There may be flexibility in the location to absorb delay relative to sectors with demand projected to exceed capacity when assigning delay to aircraft to maintain demand below capacity. There are two objectives considered by the ATSP and AOCs when selecting the location to absorb delay. The first is the ranking of the flight in the queue. A downstream location close to the constraint would be preferred in this case since an aircraft may lose its place in the queue by absorbing delay upstream further away from the constraint. The second consideration is the
impact of the flight on congestion. If the flight absorbs delay downstream closer to the constraint then the ATSP may have insufficient flexibility in terms of options to maintain demand below capacity.

To model the differing ATSP and AOCs behavior, a transition boundary is constructed at a threshold distance from the constraint along the path of an aircraft. The transition boundary is used for planning delay absorption based on the current location of the aircraft. Upstream of this transition boundary, more emphasis is placed on preserving the place of this aircraft in the flight ranking list. The impact of this flight on congestion is given increased weight downstream of the transition boundary. So by placing the transition boundary further from the constraint, congestion is emphasized, while moving the transition boundary closer to the constraint increases emphasis on preserving the ranking of flights in the queue. The transition boundary is placed at sector edges. If the transition boundary falls within the sector then the boundary is moved outwards from the constraint to align with the sector edges.

A Mixed Integer Program (MIP) described in the Appendix minimizes the deviation from a desired delay distribution over a sequence of sectors for a flight. The delay distribution is represented by the percentage of delay desired in each sector. The MIP model first minimizes delay and then moves to a second stage that minimizes the deviation from the preferred distribution, representing either the ATSP or AOC behavior, as described next, while keeping delay at the minimum level. For the ATSP the use of a location to absorb delay corresponds to Block 4 in Fig. 1 while the AOCs generate a location to absorb delay preference in Block 2 in Fig. 1.

1. **ATSP Location to Absorb Delay**

   When a flight is inside the transition boundary, the ATSP prefers to absorb delay upstream which is implemented as ground delay for flights on the ground and airborne delay in the current sector for flights that are airborne. If not all of the delay can be absorbed in the current sector, then a portion of the delay is incrementally moved downstream until demand is reduced below capacity. The MIP model minimizes the deviation from the upstream delay distribution as described in the Appendix. When a flight is outside the transition boundary, the ATSP prefers a more downstream distribution where all of the delay is absorbed in the first sector bordering the transition boundary on the outside.

2. **AOC Location to Absorb Delay**

   The AOC preference for location to absorb delay relative to a constraint is similar to the ATSP behavior except the transition boundary is positioned closer to the constraint. This models the observed behavior of the airlines preferring to postpone the absorption of delay, particularly when uncertainty is high for a flight that is still far from the constraint. The desired delay distribution inside the transition boundary remains upstream but outside the transition boundary the behavior is shifted to prefer a downstream distribution close to the constraint. The AOC selects the downstream location relative to the constraint while the ATSP selects the downstream location relative to the boundary. The AOC preference is sent to the ATSP and the ATSP in the current model always incorporates the AOC preference when making delay decisions. Future research may consider the ATSP overriding the AOC preference.

III. **Results**

   Initial simulation results are presented for collaboration between the ATSP and AOCs to solve a situation where demand is projected to exceed capacity for sectors in Cleveland Center (ZOB), which has significant east-west flow between west coast, Chicago, and New York airports. An artificial reduction in en route airspace capacity due to severe convective weather on June 19, 2007 is used to reduce capacity below demand as shown in Fig. 3. Normally a weather system of this magnitude would require coordination between the Air Traffic Control System Command Center (ATCSCC) and several Air Route Traffic Control Centers (ARTCCs). However, for this analysis the focus was only on weather affecting ZOB. The capacity for the sectors listed in Table 2 were reduced during 5:00 PM Eastern Daylight Time (EDT) to 7:00 PM EDT on June 19, 2007. This caused demand to exceed capacity a total of 1,953 aircraft-minutes based on aircraft following their nominal trajectories and without any TFM initiative. The total is the product of the number of aircraft above the capacity of the sector and the time duration when capacity is exceeded.

A discrete event simulation environment was used for this study. The simulation includes models for the AOCs and ATSP as agents that collaborate through a messaging system to maintain demand below capacity. The simulation is time-stepped by fixed time increments of one-minute duration. The platform leverages the capabilities of the Future ATM Concepts Evaluation Tool (FACET)\textsuperscript{18} for modeling sectors and

<table>
<thead>
<tr>
<th>Sector</th>
<th>MAP Value</th>
<th>Reduced Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZOB79</td>
<td>19</td>
<td>6</td>
</tr>
<tr>
<td>ZOB77</td>
<td>16</td>
<td>5</td>
</tr>
<tr>
<td>ZOB74</td>
<td>16</td>
<td>6</td>
</tr>
<tr>
<td>ZOB59</td>
<td>16</td>
<td>6</td>
</tr>
<tr>
<td>ZOB57</td>
<td>16</td>
<td>5</td>
</tr>
</tbody>
</table>
aircraft trajectories.

The simulation was set up with the parameters described in Table 3. The AOC route preference coefficients were selected assuming that fuel burn ($\alpha_1$) is the most important factor for all airlines. Legacy airlines that use a hub-and-spoke network structure consider the impact of delays on connecting passengers ($\alpha_2$) while low-cost and small airlines that operate mostly point-to-point service do not consider this factor. Customer satisfaction ($\alpha_4$) is considered by all airlines but is given relatively less weight by legacy and low-cost carriers. NAS impacts ($\alpha_3$) is not emphasized for all airline types, but relatively more for legacy and low-cost carriers that operate at more congested airports and in more congested airspace and hence need to consider to get their preferences granted. The route preference coefficients sum to one. Thus the algorithm behavior reported in this case study is limited to the parameter values in Table 3, while a more comprehensive analysis will be reported in follow on papers.

ASDI flight data were used in FACET to extract 5,422 flights that are projected to enter the constraint, other ZOB sectors, Chicago center (ZAU) sectors, or New York center (ZNY) sectors during the time demand is projected to exceed capacity (5:00 PM EDT to 7:00 PM EDT). A total 2,334 to 2,658 flights were projected to contribute to the congestion even though a flight may not incur any delay. The airspace where demand exceeds sector capacity, listed in Table 2, may enlarge in time and space thus capturing more flights than originally projected. This is the reason why the number of flights projected to contribute to the congestion is not the same for all simulation runs. The remaining flights are background traffic that were never projected to enter a congested sector during a time when the sector was congested.

The collaboration scheme was varied during the simulations, as reported in Fig. 3-5. A baseline without AOCs sending preferences and without AOCs taking responsibility for decisions is first considered. This is followed by examining the impact of each of the three types of preferences separately and combined. The AOCs are not taking responsibility for flight route and delay decisions when testing for the effects of the preferences. The last collaboration scheme considered is the combined effect of all preferences and the AOCs taking responsibility for flight route and delay decisions. For each of these collaboration schemes all AOCs participate in the collaboration while general aviation and other flights are not participating.
One metric that is used to quantify the impact of collaboration scheme is the unsolved remaining demand that exceeds capacity. This metric is based on aircraft following their trajectory with a TFM initiative in place. The metric is calculated by comparing the observed (not predicted) demand and capacity for each sector at each time increment in the simulation and if the observed demand exceeds capacity then the metric is increased by the difference between the observed demand and capacity for the sector multiplied by the time duration. The problem of sector demand exceeding capacity is considered solved if this metric equals zero. Larger values of this metric indicate that less of the problem has been solved. This metric considers both the sectors that were congested and other sectors that have become congested because of the ATSP and AOC TFM actions.

Allowing AOCs to send a location to absorb delay preference seems to increase the unsolved remaining demand that exceeds capacity as shown in a box plot in Fig. 4 based on 15 simulation runs. The whiskers on the boxes represent the maximum and minimum results. The bottom and top of the boxes represent the first and third quartiles, respectively, while the median is represented in between. When the AOCs only send flight ranking or only send route ranking preferences the unsolved remaining demand that exceeds capacity is not increased over the baseline ATSP behavior with no preferences. However, the combined effect of all three preferences can be worse than only route ranking preferences the unsolved remaining demand that exceeds capacity is not increased over the baseline.

The problem of sector demand exceeding capacity is considered solved if this metric equals zero. Larger values of this metric indicate that less of the problem has been solved. This metric considers both the sectors that were congested and other sectors that have become congested because of the ATSP and AOC TFM actions.

Table 3. Parameters in CTFM simulations.

<table>
<thead>
<tr>
<th>Agent</th>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ATSP</td>
<td>Detect demand exceeding capacity interval</td>
<td>15 minutes</td>
<td>The interval at which the ATSP evaluates if demand exceeds capacity. If demand is projected to exceed capacity then the process to generate a traffic flow management plan is initiated.</td>
</tr>
<tr>
<td>ATSP</td>
<td>Projection time</td>
<td>120 minutes</td>
<td>The projected time in the future measured from the current simulation time that the ATSP estimates demand and capacity for a sector.</td>
</tr>
<tr>
<td>ATSP</td>
<td>Delay map bin size</td>
<td>15 minutes</td>
<td>Delay observations are grouped and averaged for a 15 minute bin size.</td>
</tr>
<tr>
<td>ATSP</td>
<td>Boundary for responsibility change</td>
<td>150 nm from constraint</td>
<td>The minimum distance measured from the sectors with projected demand exceeding capacity to the location where the AOCs have responsibility for flight and route decisions.</td>
</tr>
<tr>
<td>ATSP</td>
<td>Route switch threshold</td>
<td>10 minutes</td>
<td>The $P_{ATSP}$ parameter.</td>
</tr>
<tr>
<td>AOC</td>
<td>Route preferences coefficients</td>
<td></td>
<td>Legacy airlines: $\alpha_1=0.5$, $\alpha_2=0.2$, $\alpha_3=0.15$ \ Low-cost airlines: $\alpha_1=0.8$, $\alpha_2=0.0$, $\alpha_3=0.1$ \ Small airlines: $\alpha_1=0.8$, $\alpha_2=0.0$, $\alpha_3=0.2$, $\alpha_4=0.0$</td>
</tr>
<tr>
<td>ATSP</td>
<td>Location to absorb delay transition boundary</td>
<td>400 nm</td>
<td>Distance from the congested sectors where a preference for a more downstream distribution is transitioned to a more upstream distribution.</td>
</tr>
<tr>
<td>AOC</td>
<td></td>
<td>200 nm</td>
<td></td>
</tr>
</tbody>
</table>

One metric that is used to quantify the impact of the collaboration scheme is the unsolved remaining demand that exceeds capacity. This metric is based on aircraft following their trajectory with a TFM initiative in place. The metric is calculated by comparing the observed (not predicted) demand and capacity for each sector at each time increment in the simulation and if the observed demand exceeds capacity then the metric is increased by the difference between the observed demand and capacity for the sector multiplied by the time duration. The problem of sector demand exceeding capacity is considered solved if this metric equals zero. Larger values of this metric indicate that less of the problem has been solved. This metric considers both the sectors that were congested and other sectors that have become congested because of the ATSP and AOC TFM actions.

Allowing AOCs to send a location to absorb delay preference seems to increase the unsolved remaining demand that exceeds capacity as shown in a box plot in Fig. 4 based on 15 simulation runs. The whiskers on the boxes represent the maximum and minimum results. The bottom and top of the boxes represent the first and third quartiles, respectively, while the median is represented in between. When the AOCs only send flight ranking or only send route ranking preferences the unsolved remaining demand that exceeds capacity is not increased over the baseline ATSP behavior with no preferences. However, the combined effect of all three preferences can be worse than only the location to absorb delay preference as illustrated by the outlier of greater than 100 aircraft minutes of unsolved remaining demand that exceeds capacity. In general, the ATSP model was too generous in granting AOC preferences and the ATSP model may need to be refined to reject some of the AOC preferences so that demand can be reduced below capacity. For example, the location the absorb delay preference is never rejected on the basis of congestion.

The unsolved remaining demand that exceeds capacity is lower when the AOCs take responsibility for flight and route decisions and the ATSP considers all three preferences as compared to only the AOCs sending the three preferences. This is because the AOCs rely on the delay map when making route and delay decisions. The delay map is generated without switching to alternative routes and without considering AOC preferences. The delay map instead includes the ATSP desired behavior of ranking airborne flights higher than flights on the ground which can lead to more ground delays and reduce the likelihood of airborne demand exceeding airspace capacity. Alternatively, when the ATSP makes decisions, the ATSP accepts AOC flight ranking preferences that can modify the ATSP flight ranking list so that flights on the ground can be ranked higher than flights in the air leading to too many flights being released into the airspace. For example, at 100% AOC participation, the percentage of the delay that was absorbed on the ground is 53% when the AOCs do not take route and delay decision responsibility compared to 62% when AOCs take responsibility in addition to sending preferences to the ATSP. The ATSP shared delay feedback and its use by the AOC model were effective in introducing sufficient consideration of NAS impacts into the AOCs.
decision making to make their actions effective at not worsening the congestion. Future research will test more aggressive AOC behavior.

![Simulation results for unsolved remaining demand that exceeds capacity.](image)

Figure 4. Simulation results for unsolved remaining demand that exceeds capacity.

Figure 5 shows the average delay per aircraft as measured relative to the filed flight plan. The averaging is done over flights that were projected to contribute to the demand that exceeds capacity of the congested sectors. The average delay per aircraft is approximately 5 minutes per aircraft for the baseline without preferences and the cases where flight ranking preferences only and route ranking preferences only are being considered by the ATSP. When the AOCs send a location to absorb delay preference only this reduces the average aircraft delay from approximately 5 minutes per aircraft to approximately 3.5 minutes per aircraft compared to the baseline. Considering all three preferences simultaneously reduces the average delay further to 3 minutes per aircraft. Finally, allowing AOCs responsibility for flight route and delay decisions in addition to sending all three preferences increases average delay from approximately 3 minutes up to 3.5 minutes per aircraft compared to the case of only sending the three preferences. While AOC delay savings are observed in all cases, the effect of AOCs making decisions independently is an increase in delay relative to using preferences only, which is contrary to AOC objectives. This is due to modeling the AOCs following the ATSP delay map very closely. Future research will investigate more realistic AOC modeling to select actions independent from the ATSP delay map.

Delay per passenger in Fig. 6 show trends similar to the average delay per aircraft in Fig. 5 but the magnitude of the passenger delay reduction is larger than the aircraft delay reduction when the location to absorb delay preference is sent as compared to the baseline. For the baseline with no preferences the average delay per passenger is approximately 5 minutes per passenger which is reduced to approximately 3 minutes per passenger when the AOCs send a location to absorb delay preference only and is further reduced to approximately 2.5 minutes per passenger when all three preferences are considered. This reflects the use of load factors explicitly in the AOC preferences.

In addition to AOC delay savings, flight ranking and route ranking preferences were being granted but their effect on the congestion, aircraft delay, and passenger delay is minimal. The average difference in aircraft positions in a queue based on the flight ranking preferences that were granted and the FCFS queue is approximately 5.4 positions. The FCFS difference metric is absolute and is based on relative differences in the queue and not any time metric. For the route ranking preference, approximately 17% of the flights that sent a route ranking preference were granted their preferred route.
Figure 5. Simulation results for average delay per aircraft.

Figure 6. Simulation results for average delay per passenger.
IV. Conclusion

This paper presented algorithms that capture key differences between the air traffic service provider and users in making decisions about flight route and delay, in the context of a new collaboration scheme that provides users more responsibility and preferences. When the service provider has responsibility near the sectors with demand projected to exceed capacity the service provider has complete knowledge of demand and capacity for each sector since flight decisions are incremental and one at a time. This complete knowledge allows the service provider to reduce demand below capacity if the demand is not excessive. However, when the users have responsibility they have no information regarding the intent of other users. Instead they rely on air traffic service provider feedback on aggregate sector counts and expected delay to reduce demand below capacity. Using this aggregate feedback, the users compute a utility function for ranking alternative routes of a flight considering both airspace congestion to increase air traffic service provider acceptability and user impacts such as fuel burn, connectivity, and on time performance. Conversely, a route’s impact on airspace congestion is the primary factor considered by the air traffic service provider when switching a flight to an alternate route. Only if two routes are equally congested or uncongested then the service provider considers the route preference sent by the users. In addition the users rank their flights according to the number of passengers and select a more downstream location to absorb delay relative to the service provider.

The behavior of the collaboration algorithms were demonstrated using a scenario for Cleveland center with a reduction in en route sector capacity due to severe convective weather. Initial results indicated that allowing users to send preferences, particularly the location to absorb delay preference, increases the size of the unsolved remaining demand exceeding capacity. In part this is due to the service provider model being too generous in granting user preferences. For example, the service provider did not reject the location to absorb delay preference. On the other hand, the service provider’s shared delay feedback and its use by the users was effective in introducing sufficient consideration of congestion impacts into the user decision making to make their independent decisions effective at not worsening the congestion. Results also indicated that both aircraft delay and passenger delay were reduced when allowing the users to send preferences with passenger delay being reduced more than aircraft delay. These insights will be used in future research to refine the user and service provider models. For example, the user model may need to be less reliant on the air traffic service provider desired behavior through delay feedback to be more realistic in representing independent user actions and the air traffic service provider model may need to be refined so that more user preferences are rejected.

Appendix

A mixed integer programming (MIP) model is described that minimizes aircraft delay subject to a location to absorb delay preference. The solution follows a 3-stage process. In the first stage the minimum delay solution is obtained. The choice of airborne or ground delay in the first stage is based on a preference. The second stage consists of modifying the minimum airborne delay solution to account for airline preferences for airborne delay. If no solution is found for the stage 1 minimize delay problem, which would only occur when minimizing airborne delay, then stage 3 is invoked that minimizes the time-weighted demand that exceeds capacity considering both ground delay and airborne delay.

The MIP model makes delay decisions for one flight and does not attempt to perform system-wide minimization across all flights. This enforces the prioritization of flights in the same order as the ranked list of flights. Currently this model is only run by ATSP agents but extensions to the AOC delay decisions could be considered.

A. Stage 1 Model

1. Minimize Delay Objective

In the first stage, the objective is to minimize delay without consideration for airline preferences as shown in Eq. (12).

\[
\min \sum_j D_j + D_G
\]  \hspace{1cm} \text{(12)}

where \(D_j\) is a decision variable representing the delay to be absorbed in sector \(j\) and \(D_G\) is a decision variable representing the delay to be absorbed on the ground. For the stage 1 model the delay to assign for each sector is calculated so that total delay is minimized and sectors with demand equal to or exceeding capacity are avoided. Equations (12)-(22) define the first stage of the model. Decision variables and coefficients appear on the left hand side and constants on the right hand side of each of these equations.
2. Air/Ground Delay

If an aircraft is on the ground and ground delay is preferred, then airborne delay is constrained to zero as shown in Eq. (13).

\[ \sum_{j} D_j = 0 \quad (13) \]

Conversely, if an aircraft is airborne or airborne delay is preferred then an explicit constraint to prevent ground delay is added in Eq. (14). Since the preference is either for ground delay or airborne delay only one of Eq. (13) or Eq. (14) shows up in the formulation.

\[ D_G = 0 \quad (14) \]

Ground delay is also used to specify the entry time to the first sector in Eq. (15). By convention the unimpeded departure time occurs at a time of zero:

\[ D_G - t_{0,entry} = 0 \quad (15) \]

where \( t_{0,entry} \) is a decision variable representing the entry time to the first sector along the route.

3. Sector Entry Time Constraint

The entry time to a sector is constrained to the exit time from the previous sector as shown in Eq. (16).

\[ t_{entry}^{j} - t_{exit}^{j-1} = 0 \quad \text{for } j=2,\ldots,m \quad (16) \]

where \( t_{entry}^{j} \) and \( t_{exit}^{j} \) are decision variables representing entry and exit times from sector \( j \) respectively and \( m \) is the total number of sectors for which delay will be applied. The first sector entry time is constant.

4. Time in Sector Constraint

Equation (17) sets the decision variable representing exit time from a sector based on the entry time, delay absorbed, and the minimum traversal time through the sector.

\[ t_{exit}^{j} - t_{entry}^{j} - D_j = ETE_j \quad \text{for } j=1,\ldots,m \quad (17) \]

where \( ETE_j \) is a constant representing minimum en route traversal time through sector \( j \).

5. Time of Sector Entry or Exit Relative to Congested Sector Time Start or End

Binary decision variables are introduced to determine whether the time of entry to a sector is earlier or later than the time a sector has demand equal to or exceeding capacity. The first of these equations, Eq. (18), specifies that the time of entry can be earlier or later than the demand that exceeds capacity but not both. There may be multiple time periods that demand exceeds capacity for a sector. Each of these time periods where demand exceeds capacity is assigned an index \( k \).

\[ LT_{j,k}^{entry,start} + GT_{j,k}^{entry,start} = 1 \quad \text{for } j=1,\ldots,m \quad k=1,\ldots,n_j \quad (18) \]

where \( LT_{j,k}^{entry,start} \) is a decision variable that equals 1 if the time of entry to sector \( j \) is less (earlier) than the start time of time period where demand exceeds capacity \( k \) for sector \( j \), and 0 otherwise. The first subscript for sector \( j \) refers to the first superscript of entry and similarly the second subscript for demand exceeding capacity \( k \) refers to the second superscript for start. Another decision variable, \( GT_{j,k}^{entry,start} \), equals 1 if the time of entry to sector \( j \) is greater (later) than the start time of demand exceeding capacity \( k \), and 0 otherwise. \( n_j \) is a constant representing the count of demand-to-capacity imbalances in sector \( j \). Corresponding to Eq. (18), Eqs. (19) and (20) are used to set the decision variables for the relative times:

\[ t_{entry}^{j} + r_{j,k}^{start} LT_{j,k}^{entry,start} \geq r_{j,k}^{start} \quad \text{for } j=1,\ldots,m \quad k=1,\ldots,n_j \quad (19) \]

\[ t_{entry}^{j} - M GT_{j,k}^{entry,start} \leq r_{j,k}^{start} \quad \text{for } j=1,\ldots,m \quad k=1,\ldots,n_j \quad (20) \]

where \( r_{j,k}^{start} \) is a constant representing the start time of demand exceeding capacity \( k \) for sector \( j \) and \( M \) is an arbitrary number larger than \( t_{entry}^{j} \). Equations (18) to (20) are for the time of entry to a sector relative to an imbalance start. Similar equations are required for three additional conditions: time of entry to a sector relative to an imbalance end, time of exit from a sector relative to an imbalance start, and time of exit from a sector relative to an imbalance end.
6. Avoid Time When Demand Exceeds Capacity

To avoid the time when demand exceeds capacity a constraint is added in Eq. (21) that specifies that the flight must exit the sector before the start time or enter after the time when demand exceeds capacity.

\[ LT_{j,k}^{exit, start} + GT_{j,k}^{entry, end} = 1 \quad \text{for } j=1,\ldots,m \quad k=1,\ldots,n_j \]  

(21)

7. Delay Increment

It is possible that the model introduces delay in increments that are too short for the controller to effectively increment. For example, a one-minute delay may be feasible solution to the model but not reasonable. To account for this a minimum delay increment is introduced in Eq. (22).

\[ D_j - \alpha_j d_{j}^{int} = 0 \quad \text{for } j=1,\ldots,m \]  

(22)

where \( \alpha_j \) is a constant coefficient representing the minimum increment of delay for sector \( j \) and \( d_{j}^{int} \) is a general integer decision variable that applies the increment to \( D_j \).

B. Stage 2 Model

Solving the stage 1 model defined in Eqs. (12) to (22) yields the minimum total delay \( D_{total}^* \) if the problem is feasible. The stage 3 model in the next section is used when a solution cannot be found to the stage 1 model. However, there may be alternative solutions as to where to allocate this delay to better accommodate airline preferences. For this model we minimize the difference between the airlines preferred location to absorb delay and a feasible delay distribution. For the stage 2 model no ground delay is used so the constraint in Eq. (14) is enforced.

Equation (23) first specifies that the delay should not be increased above the minimum delay.

\[ D_{total}^* - \sum_j D_j = 0 \]  

(23)

where \( D_{total}^* \) is a constant representing the minimum total delay obtained from the stage 1 solution. The desired proportion of this total minimum delay is then applied to a decision variable representing the desired delay to be absorbed in sector \( j \) as shown in Eq. (24):

\[ D_j^* - p_j D_{total}^* = 0 \quad \text{for } j=1,\ldots,m \]  

(24)

where \( p_j \) is a constant representing the proportion of the total delay desired for sector \( j \) and \( D_j^* \) is a decision variable representing the desired delay to be absorbed in sector \( j \). An additional constraint is specified outside the model: the sum of \( p_j \) should equal 1.

A standard method is used in Eqs. (25) and (26) to determine the difference between the desired delay and the absorbed (or calculated) delay:

\[ D_j^* - D_j - \delta_j \leq 0 \quad \text{for } j=1,\ldots,m \]  

(25)

\[ -D_j^* + D_j - \delta_j \leq 0 \quad \text{for } j=1,\ldots,m \]  

(26)

where \( \delta_j \) is a decision variable representing the absolute value of the difference between \( D_j^* \) and \( D_j \).

The objective function in Eq. (12) is now modified to Eq. (27) to minimize the difference between the desired delay distribution and a feasible delay distribution. The constraints in the stage 2 model, Eqs. (23) to (26), are in addition to the constraints defined in Eqs. (16) to (22) for the stage 1 model.

\[ \min \sum_j \delta_j \]  

(27)

C. Stage 3 Model

The following formulation is used when a feasible solution is not found for the stage 1 model. The stage 2 model also has no solution if the stage 1 model is infeasible. The objective of the stage 3 model is to minimize the product of \( (\text{time over capacity}) \) and \( (\text{demand minus capacity}) \). In the stage 3 model the demand exceeding capacity for higher ranked aircraft is a constant so the minimization problem is the contribution of this aircraft to the magnitude of the demand exceeding capacity.

If a flight is on the ground then both ground delay and airborne delay is permitted in order to consider the largest possible solution space. If both ground delay and airborne delay are permitted then neither Eq. (13) nor Eq. (14) is enforced. If a flight is airborne then ground delay is not possible and Eq. (14) is enforced.
For the stage 3 model, the Eq. (21) constraint in the stage 1 model is deleted and replaced with Eq. (28) to allow demand to exceed capacity which is then penalized in the objective function.

\[ LT_{j,k}^{exit,start} + GT_{j,k}^{entry,end} + I_{j,k} = 1 \quad \text{for } j=1,...,m \quad k=1,...,n_j \]  

(28)

where \( I_{j,k} \) is a decision variable that equals 1 if demand exceeding capacity during time period \( k \) in sector \( j \) cannot be avoided, and 0 otherwise. The objective function for the stage 1 model defined in Eq. (12) is replaced with the stage 3 objective function minimizing the time-weighted demand exceeding capacity defined in Eq. (29)

\[
\min \sum_j \sum_k (t_{j,k} q_{j,k} - c_{j,k}(t_{j,k}^{end} - t_{j,k}^{start}))
\]

(29)

where \( q_{j,k} \) is a constant representing the demand, \( c_{j,k} \) is a constant representing the sector capacity, \( t_{j,k}^{end} \) is the end time, and \( t_{j,k}^{start} \) is the start time all for sector \( j \) during time period \( k \) when demand exceeds capacity.

Acknowledgments

This research was funded by NASA under contract NNA07BB35C. The authors would like to thank Jose Garcia-Chico, Sharon Woods, Brendan Lefebvre, Robert Vivona, David Karr, Tarek El-Wakil of L-3 Communications and Jason Burke and Jason Pepper of Metron Aviation Inc. for contributions to algorithm and software development.

References


