Analysis of Thunderstorm Effects on Aggregated Aircraft Trajectories

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The utility of route guidance and trajectory prediction tools in air traffic management is directly related to how well such tools anticipate pilot and controller reactions to weather. This paper presents a new method for translating weather data into patterns in aggregated aircraft trajectories. Techniques are described that limit the human and computational effort required to analyze large sets of data and enable formulation and discovery of mathematical relationships among large numbers of weather- and flight plan-related variables. The method is used to examine the effects of thunderstorms on aggregate aircraft operations near Atlanta in the spring and summer of 2007. Measures of precipitation intensity and storm cell height were related to aircraft positions over a period of 40 days. A mathematical model of the relationship between precipitation intensity, storm cell height, flight level, and airspace occupancy was constructed using multivariate adaptive polynomial spline regression. Explanatory power was lost when aircraft altitude and storm cell height readings were combined into a measure of their difference. Precipitation intensity contributed surprisingly little discriminatory power to the built model. Aircraft sought to avoid airspace within 5 km of storm activity, rerouting to airspace 10 km to 20 km and farther from the storm.

I. Introduction

Several efforts in air traffic management research propose methods for constructing decision-support tools to increase the safety and efficiency of the air traffic control (ATC) system. As Kuchar et al. [1] have noted, these tools will be accepted by the operators of the ATC system only if they are “aligned with operator mental models”. Currently, understanding of how the risks created by thunderstorm activity are evaluated and how mitigation strategies are chosen by pilots and ATC system operators is limited. Increasing that understanding would also help determine what constitutes safe and efficient operations.

Past efforts to use empirical data to model how available weather data “translate” into aggregated aircraft trajectories have classified aircraft into two groups: those that penetrate severe weather and those that deviate around it. Weather data have then been correlated to this classification. Separating penetrating aircraft from deviating aircraft has proved difficult, even when given full trajectory and weather information, requiring human analyst input. This has limited the sizes of data sets considered, reducing the confidence placed in obtained results and making it difficult to add explanatory variables or separately consider combinations of variables.

This paper presents a new method for translating weather data into patterns in aggregated aircraft trajectories. This new method reduces the human and computational effort required to analyze large sets of data and enables the formulation and discovery of mathematical relationships among large numbers of weather- and flight plan-related
variables. This method is demonstrated by examining aggregated aircraft operations in the spring and summer of 2007 in the airspace around Atlanta Hartsfield–Jackson International Airport (ATL), controlled by the Atlanta Air Route Traffic Control Center (ZTL) and the Atlanta Terminal Radar Approach Control (TRACON).

II. Background

A handful of research efforts have examined the translation problem. In early 1999, Rhoda and Pawlak [2] studied a great variety of weather data, including information regarding precipitation intensity using radar vertically integrated liquid (VIL) measurements. Data were collected during 63 h of operations at the Dallas–Fort Worth TRACON. A human analyst bifurcated the set of aircraft trajectories into sets of aircraft deviating around and penetrating storm activity. Radar reflectivity and VIL measures were shown to be relevant to classification. Kuchar et al. [1] studied a similar data set, taken during 12 h of operations at the Dallas-Fort-Worth TRACON and focusing on 353 penetration trajectories. The authors found that all aircraft avoided VIL level 6 activity, but that as VIL levels decreased, gradually more aircraft flew through storm activity for extended periods of time.

In 2002, Rhoda et al. [3] studied enroute and terminal airspace controlled by the Memphis Air Route Traffic Control Center (ARTCC) and the Memphis TRACON, respectively. The authors noted differences between the behaviors of deviating aircraft in enroute vs terminal airspace. The authors found that aircraft in the terminal area were more likely to penetrate storm activity. In addition, the study showed that a number of aircraft flying in enroute airspace were flying above storms recording VIL levels of 3 or higher. Six days of operations, representing 43.5 h of relevant data were analyzed, but the conclusions noted that: “A much larger dataset of 3-D enroute storm encounters should be compiled and analyzed”.

DeLaura and Evans [4] examined 472 trajectories from the Indianapolis ARTCC and 539 trajectories from the Cleveland ARTCC. Based on this larger data set, they concluded that “the primary factor in weather-related deviations is the height of the storm relative to the flight altitude.” An automated procedure for splitting aircraft trajectories into penetrations and deviations was introduced, but it was still considered necessary to have a human analyst check and edit results. DeLaura and Evans found that VIL information alone was a poor predictor of deviation/penetration decision making. Even for the most accurate models created, combining multiple sources of weather data, classification errors ranged from 19 to 26%. An examination of 218 deviation trajectories revealed aircraft flew along the contours of VIL levels. However, the authors warn: “It is important to note that one must use caution in interpreting the avoidance analysis data. It provides insights into pilot behavior that must be confirmed by analysis of a larger dataset.”

A previous work by DeLaura and Allan [5] found that relying on precipitation intensity measures alone “resulted in route selection guidance that, in some cases, was too conservative and closed routes that were passable and in other cases, declared as passable regions that pilots consistently avoided.” The utility of route guidance decision-support and trajectory prediction tools is directly related to how well such tools are able to anticipate pilot and controller reactions to observations of different patterns of weather data. The ability to predict these reactions is limited today. Past efforts to increase prediction capabilities have been hampered by the limited data sets that these efforts considered.

III. Methodology and Data

To improve analytical capabilities, the present work developed and analyzed large data sets by shifting the frame of reference from individual aircraft trajectories to individual weather patterns. The method applies to terminal area airspace as easily as it does to enroute airspace. Detailed precipitation intensity, storm cell height, and aircraft position data were collected for 40 days during the spring and summer of 2007. Analysis considered a variety of different combinations of weather data measurements and their impacts at different flight levels. The sizes of the data sets analyzed and uncertainty regarding the distributions of derived summary data led to the use of graphical representations of data and nonparametric tests of assumptions and model building.

Data were collected from airspace in the vicinity of Atlanta, Georgia, roughly defined as the area between 32 and 35 degrees North latitude, and 82 and 85 degrees West longitude. This airspace includes ATL and is primarily controlled by ZTL and Atlanta TRACON. This airspace was chosen because of the large volume of arriving and departing aircraft, and because thunderstorms are a problem in this area during the spring and summer months.

The corridor integrated weather system (CIWS) provided information regarding weather conditions. CIWS is an aviation weather system managed by the Massachusetts Institute of Technology Lincoln Laboratory [6]. Precipitation
intensity was measured using the VIL scale. Storm cell height was measured using high-resolution radar echo tops. VIL and echo top information, when available, were obtained every half hour for 1 km by 1 km cells within the airspace of interest. Certain patterns in the weather data were infrequently observed, including low echo top heights paired with high VIL levels and vice versa.

Aircraft positions in ZTL were provided by the Center-TRACON Automation System (CTAS). CTAS is a suite of decision-support tools developed at the NASA Ames Research Center [7], which receives ATC center radar track updates every 12 s from the Federal Aviation Administration. The positions of individual aircraft were recorded a variable number of times since aircraft spent variable lengths of time in the airspace of interest.

The detail provided by CTAS and CIWS combined with the time scale of analysis produced nearly 160 million measurements of weather data, along with over 13 million aircraft position data points. Combining CTAS data with CIWS data provided aircraft observation counts, by flight level, in particular grid cells with known weather conditions.

Analyses undertaken in this paper averaged aircraft observation counts across grid cells reporting similar weather conditions. The goal was to estimate the impacts on aircraft observation counts of the different weather conditions. This would reveal, for instance, the conditions through which pilots always or occasionally avoided flying. Several dependencies relate aircraft observation counts from individual grid cells. Two of the clearest dependencies involve aircraft routing procedures and seasonality; counts were higher in grid cells on standard routes, as well as during times when large banks of aircraft arrive or depart. The great majority of the dependencies relating individual aircraft counts are insignificant when comparing different values of average observation counts. Statistical tests can show if biases do exist, and well-known techniques can be used to correct these biases. Considering 40 days of data minimized the impacts of many dependencies based on temporal proximity of data points.

**IV. Results**

Table 1 shows averaged aircraft observation counts across grid cells reporting similar weather conditions. It is again worth noting that relatively few grid cells were observed reporting high VIL levels and low echo tops, as well as low VIL levels and high echo tops. This increases the variability of data corresponding to such conditions.

The data from Table 1 support two meaningful results identified in past research. There is no clear dividing line between what pilots do and do not fly through today. For instance, pilots sometimes, but not always, avoid flying through storms reporting a VIL level of 2 and an echo top of 30–35,000 ft. Also, VIL information alone is much less meaningful than the combination of VIL and radar echo top data. There is significant variability in the numbers of aircraft observed per grid cell for any given VIL level.

Intuitive assumptions are that fewer planes will be observed in an area of airspace as VIL levels or radar echo tops increase. These assumptions were tested using Page’s L test [8]. This test was chosen for its ability to prove, at a given confidence level, whether the data supported hypothesized trend relationships, without making assumptions regarding the distributions of the data. Data presented in Table 1 were first matched by echo top to test the assumption regarding VIL levels, and then vice versa. The test statistic for Page’s L Test is defined as

\[ \sum_{i=1}^{n} Y_i \sum_{j=1}^{k} r_{ij} \]

<table>
<thead>
<tr>
<th>Echo top height ft</th>
<th>VIL level 0</th>
<th>VIL level 1</th>
<th>VIL level 2</th>
<th>VIL level 3</th>
<th>VIL level 4</th>
<th>VIL levels 5, 6</th>
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<tbody>
<tr>
<td>≤ 5000</td>
<td>42</td>
<td>60</td>
<td>62</td>
<td>45</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5–10,000</td>
<td>36</td>
<td>56</td>
<td>76</td>
<td>186</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10–15,000</td>
<td>81</td>
<td>90</td>
<td>85</td>
<td>57</td>
<td>11</td>
<td>0</td>
</tr>
<tr>
<td>15–20,000</td>
<td>87</td>
<td>86</td>
<td>76</td>
<td>75</td>
<td>81</td>
<td>68</td>
</tr>
<tr>
<td>20–25,000</td>
<td>102</td>
<td>84</td>
<td>69</td>
<td>65</td>
<td>63</td>
<td>69</td>
</tr>
<tr>
<td>25–30,000</td>
<td>105</td>
<td>68</td>
<td>56</td>
<td>50</td>
<td>52</td>
<td>48</td>
</tr>
<tr>
<td>30–35,000</td>
<td>97</td>
<td>67</td>
<td>40</td>
<td>32</td>
<td>31</td>
<td>28</td>
</tr>
<tr>
<td>≥ 35000</td>
<td>128</td>
<td>70</td>
<td>33</td>
<td>16</td>
<td>15</td>
<td>11</td>
</tr>
<tr>
<td>All echo top heights</td>
<td>45</td>
<td>79</td>
<td>61</td>
<td>45</td>
<td>38</td>
<td>23</td>
</tr>
</tbody>
</table>
Table 2 Results of Page’s L tests.

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Test statistic</th>
<th>Critical value (α = 0.05)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIL level</td>
<td>700</td>
<td>625</td>
</tr>
<tr>
<td>Echo top</td>
<td>936</td>
<td>1037</td>
</tr>
</tbody>
</table>

Table 3 Results of $L_D$ test and further Page’s L tests.

<table>
<thead>
<tr>
<th>Test statistic</th>
<th>Critical value (α = 0.05)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_D$ test</td>
<td>0 70</td>
</tr>
<tr>
<td>Page’s L test</td>
<td>468 736</td>
</tr>
<tr>
<td>VIL levels ≥3</td>
<td></td>
</tr>
<tr>
<td>Page’s L test</td>
<td>468 736</td>
</tr>
<tr>
<td>VIL levels ≤2</td>
<td></td>
</tr>
</tbody>
</table>

where $n$ is the number of levels of the test criterion, 6 when testing the assumption regarding VIL levels, $k$ is the number of matched sets of data, 8 in this case since there are 8 ranges of echo top heights considered, $Y_i$ is the assumed ranking of the $i$th level of data, so that $Y_1$ was set to 6 to indicate that the first lowest range of VIL levels (VIL level 0) would have the sixth lowest (i.e., the highest) values for aircraft observations per grid cell, $r_{ij}$ is the actual rank of the $i$th data point within the $j$th match set of data, so that $r_{21}$ was set equal to 5. Test statistics, presented in Table 2, were compared to critical values taken from Page’s original work. Where values calculated from data exceed critical values, there is evidence of a trend in the data.

With a confidence level of 95%, the results support the hypothesis that as VIL levels increase, average numbers of aircraft observations decrease. The same cannot be said for echo tops.

Intuition may suggest that echo tops are important only for significant storms where pilots are willing to fly above, but not through, the precipitation. The $L_D$ test [9], an extension of Page’s L test, was used to test this hypothesis. This procedure involved splitting the data into two subsets: data related to VIL levels 2 or lower and 3 or higher. Page’s L tests were run for each subset of the data. The absolute value of the difference of the two test statistics forms the test statistic for the $L_D$ test. Results, presented in Table 3, show absolutely no difference between areas reporting VIL levels $\leq 2$ and $\geq 3$ in terms of their agreement with the hypothesis that as echo top levels increase, observations of planes in an airspace decrease.

Results presented in Tables 2 and 3 related to echo top heights do not imply that this information is irrelevant, but rather there is an absence of proof to support the specific hypothesis that as echo top heights increase, aircraft counts monotonically decrease. Table 1 clearly shows that echo top heights are meaningful in pilot and controller decision making.

V. Differentiating Impacts by Flight Level

Previous studies predicted pilot behavior by looking at the difference between a plane’s planned flight level and radar echo tops. A question arises as to whether any explanatory power would be lost in this study if echo tops and aircraft altitudes were combined into measures of their difference.

Analysis begins, as before, with an examination of average numbers of aircraft observations in regions of airspace of a fixed size. Individual grid cells were split by altitude, creating regions set distances above and below radar echo tops measurements. Data were matched by VIL level, and distance above/below echo tops. The Friedman test [10] was used to test whether radar echo top measurements remained significant in determining average aircraft observation counts. The test statistic for the Friedman test is

$$\frac{12}{kn(n+1)} \sum_{i=1}^{n} \left( \sum_{j=1}^{k} r_{ij} \right)^2 - 3k(n+1)$$
Table 4 Results of Friedman test.

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Test statistic</th>
<th>Critical value $(\alpha = 0.05)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Echo top</td>
<td>1278</td>
<td>198</td>
</tr>
</tbody>
</table>

keeping the same notation as used previously. In this particular case $n = 10$ because 10 levels of echo top ranges were compared, and $k = 168$ because 168 combinations of VIL level and difference between aircraft altitude and echo top were considered. The value of $k$ was set quite large to ensure that the impacts of precipitation intensity and the difference between aircraft altitude and echo top were accounted for. Table 4 presents the results. It is clear from Table 4 that echo top measurements are significant, even when the differences between aircraft altitude and echo top already have been taken into account.

Based on Table 4, aircraft altitudes, echo tops, and VIL levels were all considered as analysis continued. As a first step, the distribution of aircraft by altitude under clear weather conditions was examined. Figure 1 shows the numbers of aircraft observations, categorized by altitude in areas reporting clear weather.

A baseline for aircraft counts per grid cell, broken down by flight level, was created by dividing total aircraft counts in cells reporting clear weather by the number of cells reporting clear weather. Similar values for aircraft counts per grid cell, broken down by flight level, were calculated for different combinations of weather data. These data are referred to in the current paper as measures of airspace occupancy. These data measure the proportions of times that aircraft were observed in a set volume of airspace over a set period of time. Airspace occupancies are indicative of pilot and controller weather avoidance preferences. Airspace occupancies can also be directly translated into probabilities of penetration, following the deviation/penetration framework of analysis.

Figure 2 contains a graphical representation of the comparison of airspace occupancy data. Where airspace occupancy equaled clear weather baseline airspace occupancy values, a white square is drawn. Where no aircraft were observed flying at the relevant flight level through the relevant weather conditions, a black square is drawn. Where values were between these two extremes, shading is used to show how high or low airspace occupancy values were. Dotted lines on each of the graphs in Fig. 2 indicate where the altitudes of echo top measurements equal those of flight levels under consideration. The farther above the dashed line a data point lies, the farther above the clouds airspace of interest lies. Weather conditions that were observed infrequently and flight levels with few aircraft observations under clear weather conditions were not included in Fig. 2, since limited confidence could be placed in results regarding such conditions. This explains why the different graphs of Fig. 2, considering different VIL levels, show data regarding different ranges of echo top heights.

**Fig. 1 Aircraft observations, by flight level, in clear weather.**
Fig. 2 Airspace occupancy by VIL level, echo top range, and flight level: a) VIL level 2, b) VIL level 3, c) VIL level 4+.
Figure 2 indicates that few aircraft, relative to baseline (clear weather) conditions, flew through airspace below echo tops. This was increasingly the case as VIL levels and echo top readings increased. There do seem to be significant numbers of aircraft flying at relatively low altitudes (≤9000 feet) through all levels of storm activity. This supports the findings of Rhoda et al. [3] who noted several reasons why this might be the case. Pilots have less flexibility in routing when arriving and departing. Airborne radars are more likely to be subject to ground clutter at low altitudes, corrupting weather data available to pilots. Figure 2 contains a lot of gray areas, again indicating that the classification of areas those that pilots will and will not fly through is not straightforward.

VI. Mathematical Model Building

A mathematical model relating weather data to airspace occupancy, simplifying the data contained in Fig. 2, would be useful in developing decision-support tools for ATC, especially in the areas of route guidance and trajectory prediction. Building such a model would also shed light on relationships between data sets studied here. It is not clear a priori how different data studied in the present paper are distributed or how they could be merged. Here, multivariate adaptive polynomial spline regression was used, as introduced in Kooperberg et al. [11]. This form of regression is similar to techniques using multivariate adaptive regression splines but requires less computational effort. Linear splines are used to fit the data, with no functional form for the relationship between predictor variables and output data assumed. The methodology is able to consider factor variables, such as VIL level, and interactions between predictor variables. This approach is especially well suited to the consideration of large numbers of explanatory variables that interact with each other in unknown ways.

Figure 3 shows the individual estimated relationships between VIL level, radar echo top height, the difference between flight level and echo top height, and airspace occupancy values normalized against a clear weather baseline. VIL level was restricted to the values of 2, 3, and 4 or higher (4+). The first graph of Fig. 3 shows the impact on normalized airspace occupancy values of these different VIL levels. Echo top height and the difference between flight level and echo top height are treated as continuous variables, although the analysis (based on the data shown in Fig. 2) considered only a discrete set of values of these variables. Relationships involving these variables are plotted in Fig. 3 as well.

Figure 3 illustrates that aircraft occupancy decreased as VIL level increased, confirming intuition and previous results. However, the discriminatory power of VIL information is very low. This is not a result of having correlated covariates, because analysis was based on the data shown in Fig. 2 and contained many data points where VIL levels were relatively high and echo top heights relatively low (and vice versa). The results support the finding of DeLaura and Evans [4] that the difference between aircraft altitude and radar echo top is the single most powerful explanatory variable in predicting whether a pilot will or will not deviate around a storm. Figure 3 also indicates that airspace 10 to 20,000 ft below echo tops is perceived by pilots as especially unsafe. Aircraft counts increase the farther above, and also below, echo tops one goes. By and large, increased echo top readings correlated with fewer aircraft observations, again confirming intuition. However, at the very lowest and highest ranges of echo top readings

![Fig. 3 Impacts of a) VIL level, b) echo top height, and c) the difference between flight level and echo top height on normalized airspace occupancy.](image-url)
Fig. 4 Model fitted values for airspace occupancy: a) VIL level 2, b) VIL level 3, and c) VIL level 4+.
the monotonic relationship broke down. This sheds light on why, in Section IV of the present paper, statistical testing did not prove that as echo tops increase aircraft observation counts decrease.

Figure 4 contains the fitted values for this model in the same format as Fig. 2, while Fig. 5 shows plots of residual values (the difference between actual values and fitted values). The first graph of Fig. 5 is a simple histogram of residual values, while the other graphs show residuals as a function of VIL level, echo top, and flight level minus echo top.

A quick comparison of Figs. 2 and 4 indicates that the general nature of the data to be fit has been captured. However, in Fig. 5, the distribution of residual values has a broad peak. The model is fairly likely to be off by 0.1 when predicting values between 0 and 1. Figure 5 does not reveal any obvious and correctable weaknesses in the mathematical model. It would be possible to go into much more detail regarding the model constructed and residual errors and noise. However, the goal here is only to introduce a new methodology for translation and to show that a mathematical model can be found to characterize pilots’ perceptions of safe airspace.
VII. Severe Storms and Surrounding Airspace

The work presented so far has examined the numbers of aircraft flying through different weather conditions. Conditions were implicitly assumed to be relevant to aircraft observation counts only in the precise areas where the conditions were reported. This section presents results obtained by exploring the impacts of different types of severe weather on areas of airspace different distances away.

Areas of airspace that reported VIL levels of 4 and higher and echo tops of 35,000 ft and higher were considered first and labeled as storms (The data presented in this paper do not support any strict bifurcation of airspace based on weather data; however, the “storm” areas were associated with the fewest numbers of aircraft observations.). Airspace was split into areas that were storms, that were not storms but were 1 km away from a storm, 2 km away, . . . , and more than 20 km from the nearest storm. A clear weather baseline of aircraft observation counts per grid cell per flight level was created from airspace more than 20 km from the nearest storm. The analysis then proceeded in a manner similar to that done previously. Data derived from the other categories of airspace were divided by clear weather mean count data. This information is plotted in Fig. 6, with a dotted line added to indicate the minimum height of echo top heights reported by storms. Plus signs were added to the graph to indicate when aircraft counts were higher than the clear weather baseline.

The results indicate that the most severe adverse impacts of storms on airspace occupancy are highly localized. Significant adverse impacts are largely confined within 5 km of a storm. The impacts of storm activity on surrounding sections of airspace decrease as distances between echo tops and the altitude ranges of the sections of airspace of interest increase. Sections of airspace 10 km and farther from a storm reported higher average aircraft observation counts than the clear weather baseline. Aircraft deviating around storm activity have flown through these areas. It is clear that significant numbers of pilots deviate at least as far as 20 km away from a storm, and it is impossible to tell the maximum distances that pilots choose to fly around storms. It is not clear how hard and soft constraints related to how airspace is managed today impacted the results. This analysis indicates that it would be easier to predict which areas an aircraft encountering severe weather will not fly through, as opposed to which it will fly through.

Fig. 6 Impacts of VIL level 4+, echo top 35,000+ feet conditions on surrounding airspace.
VIII. Conclusion

The primary purpose of this paper is to introduce a new methodology for evaluating how weather data translate to impacts on aggregated aircraft trajectories. This new methodology allows for the consideration of larger data sets and the discovery of more complex translation models than was the case previously.

A study focusing on airspace near Atlanta, Georgia was conducted. The results indicate that precipitation intensity, storm cell height, and aircraft altitude information are all relevant to pilot decision making. Thus data in these areas are important to aviation systems researchers particularly in the areas of route guidance and trajectory prediction. A model relating measures of relevant data to airspace occupancies was created using multivariate adaptive polynomial spline regression. Precipitation intensity contributed the least to the built model, which is surprising given the emphasis placed upon it in aviation systems research. It was shown that as precipitation intensity increases, fewer aircraft will fly through a given area of airspace. A similar, but more complex, relationship was found relating to storm cell heights. Storm cell height was a significant explanatory variable in modeling aggregate aircraft operations, even when the differences between cloud heights and aircraft altitudes had already been taken into account. Airspace occupancies were low in sections of airspace within 5 km of a severe storm, but were quite large in airspace 10 km to 20 km and farther from the nearest storm.

Future work will consider additional explanatory variables, including aircraft types as well as measures of airspace structure and complexity, in models of airspace occupancies. One issue that has not been addressed in the current paper is the temporal separation between when a pilot decides his or her trajectory and when the plane actually is observed flying around, or through, convective weather. Pilots make decisions based on one set of weather data and beliefs regarding how weather conditions will evolve, which may prove different than the actual future weather the pilots encounter. It would be worthwhile to consider reformulations of the analyses performed here that address this issue. In addition, findings related to the impacts of severe storms on surrounding airspace shown in this paper will be incorporated into mathematical models.

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References
